① $\displaystyle\int_0^1 \frac{e^{9x}}{2+4e^{9x}}\,dx$

$u = 2+4e^{9x}$

$du = 36e^{9x}\,dx$

$e^{9x}\,dx = \dfrac{du}{36}$

when $x=0$ $u=6$

$x=1$ $u=2+4e^9$

$= \dfrac{1}{36}\displaystyle\int_6^{2+4e^9} \frac{du}{u}$

$= \dfrac{1}{36}\Big[\ln|u|\Big]_6^{2+4e^9}$

$= \dfrac{1}{36}\left[\ln|2+4e^9| - \ln|6|\right]$

$= \dfrac{1}{36}\ln\left|\dfrac{2+4e^9}{6}\right|$

or $\dfrac{1}{36}\ln\left(\dfrac{1+2e^9}{3}\right)$

(2) $\displaystyle\int_0^{\frac{\pi}{4}} 4\sqrt{\tan x}\ \sec^2 x\, dx$

$u = \tan x$

$du = \sec^2 x\, dx$

When $x = 0 \quad u = 0$

$x = \frac{\pi}{4} \quad u = \tan\frac{\pi}{4} = 1$

$= \displaystyle\int_0^1 4\sqrt{u}\, du$

$= \frac{4}{5}\left[u^{5/4}\right]_0^1$

$= \frac{4}{5}\left[1 - 0\right]$

$= \frac{4}{5}$

③ $\displaystyle\int \frac{x}{\sqrt{4-x^2}}\,dx$

$$u = 4 - x^2$$
$$du = -2x\,dx$$
$$x\,dx = \frac{-du}{2}$$

$$= -\int \frac{du}{2\sqrt{u}}$$

$$= -\sqrt{u} + C$$

$$= -\sqrt{4-x^2} + C$$

④

$$\int \frac{4\,dx}{\sec x\; e^{\sin x}}$$

$$= \int \frac{4\cos x\,dx}{e^{\sin x}}$$

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x\,dx \end{aligned}}$$

$$= \int \frac{4\,du}{e^{u}}$$

$$= \int 4e^{-u}\,du$$

$$= -4e^{-u} + C$$

$$= -4e^{-\sin x} + C$$

(5) $\int \frac{x}{\sqrt{1+x}}\,dx$

Integration by Parts

	D	I
$+$	x	$\frac{1}{\sqrt{1+x}}$
$-$	1	$2\sqrt{1+x}$
		$\frac{4}{3}(1+x)^{3/2}$

$$\int \frac{x}{\sqrt{1+x}}\,dx = 2x\sqrt{1+x} - \frac{4}{3}(1+x)^{3/2} + C$$

6 $\displaystyle\int \frac{e^{-x}\,dx}{1+e^{-2x}}$

$$= \int \frac{e^{-x}\,dx}{1+(e^{-x})^2}$$

$u = e^{-x}$

$du = -e^{-x}\,dx$

$e^{-x}\,dx = -du$

$$= -\int \frac{du}{1+u^2}$$

$$= -\tan^{-1} u + C$$

$$= -\tan^{-1}(e^{-x}) + C$$

(7) $$\int_0^{\frac{\pi}{4}} \frac{5-8\sin 2x}{\cos^2 x}\,dx$$

$\boxed{\text{Recall } \sin 2x = 2\sin x\cos x}$

$$= \int_0^{\frac{\pi}{4}} \frac{5-16\sin x\cos x}{\cos^2 x}\,dx$$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{5}{\cos^2 x} - \frac{16\sin x\cancel{\cos x}}{\cancel{\cos^2 x}\cos x}\right)dx$$

$$= \int_0^{\frac{\pi}{4}} (5\sec^2 x - 16\tan x)\,dx$$

$$= \Big[5\tan x - 16\ln|\sec x|\Big]_0^{\pi/4}$$

$\boxed{\begin{array}{l}\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \Rightarrow \sec\frac{\pi}{4} = \sqrt{2} \\ \cos 0 = 1 \Rightarrow \sec 0 = 1\end{array}}$

$$= (5 - 16\ln\sqrt{2}) - (0 - 16\ln 1)$$

$$= 5 - 16\ln\sqrt{2}$$

or $5 - 8\ln 2$

8 $\displaystyle\int \frac{\ln x}{x^2}\,dx$

Integration by Parts

$$\begin{array}{c c|c c}
 & D & I & \\
\hline
(u) & \ln x & x^{-2} & (dv) \\
(du) & \frac{1}{x} & -x^{-1} & (v) \\
 & \vdots & \vdots &
\end{array}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int \frac{\ln x}{x^2}\,dx = -x^{-1}\ln x + \int x^{-2}\,dx$$

$$= -x^{-1}\ln x - x^{-1} + C$$

(9) $\int \frac{3}{x^2-25}\,dx$

$= \int \frac{3}{(x-5)(x+5)}\,dx$

Partial Fractions

$\frac{3}{(x-5)(x+5)} = \frac{A}{x-5} + \frac{B}{x+5}$

$3 = A(x+5) + B(x-5)$

Sub $x=5$: $3 = 10A$ $A = \frac{3}{10}$

Sub $x=-5$: $3 = -10B$ $B = \frac{-3}{10}$

$= \int \frac{3}{10}\cdot\frac{1}{x-5} - \frac{3}{10}\cdot\frac{1}{x+5})\,dx$

$= \frac{3}{10}\ln|x-5| - \frac{3}{10}\ln|x+5| + C$

or $\frac{3}{10}\ln\left|\frac{x-5}{x+5}\right| + C$

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$$\int \frac{5\,dx}{x^2+8x+17}$$

Complete the square

$$x^2+8x+17 = (x+4)^2+\,?$$

$$= (x+4)^2+1$$

$$= \int \frac{5\,dx}{1+(x+4)^2}$$

$u = x+4$

$du = dx$

$$= \int \frac{5\,du}{1+u^2}$$

$$= 5\tan^{-1}u + c$$

$$= 5\tan^{-1}(x+4) + c$$

(11) $$\int \frac{y^3+1}{y^3+5y^2+4y}\,dy$$

Long Division

$$\begin{array}{r} 1 \\ (y^3+5y^2+4y)\overline{)\,y^3+0y^2+0y+1} \\ -(y^3+5y^2+4y) \\ \hline -5y^2-4y+1 \end{array}$$

$$= \int\left(1 + \frac{-5y^2-4y+1}{y^3+5y^2+4y}\right)dy$$

$$y^3+5y^2+4y = y(y^2+5y+4)$$
$$= y(y+1)(y+4)$$

$$= \int\left(1 + \frac{-5y^2-4y+1}{y(y+1)(y+4)}\right)dy$$

Partial Fractions

$$\frac{-5y^2-4y+1}{y(y+1)(y+4)} = \frac{A}{y} + \frac{B}{y+1} + \frac{C}{y+4}$$

$$-5y^2-4y+1 = A(y+1)(y+4) + By(y+4) + Cy(y+1) \quad \rightarrow$$

Sub $y=0$: $1=4A$ $A=\frac{1}{4}$

Sub $y=-1$: $0=-3B$ $B=0$

Sub $y=-4$: $-63=12C$ $C=\frac{-63}{12}=\frac{-21}{4}$

$$=\int\left(1+\frac{1}{4}\cdot\frac{1}{y}-\frac{21}{4}\cdot\frac{1}{y+4}\right)dy$$

$$=y+\frac{1}{4}\ln|y|-\frac{21}{4}\ln|y+4|+C$$

(12) $f(x,y) = e^{x}\cos y + e^{-2x}\tan y$

$$\frac{\partial f}{\partial x} = e^{x}\cos y - 2e^{-2x}\tan y$$

$$\frac{\partial f}{\partial y} = -e^{x}\sin y + e^{-2x}\sec^{2} y$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left(-e^{x}\sin y + e^{-2x}\sec^{2} y\right)$$

$$= -e^{x}\sin y - 2e^{-2x}\sec^{2} y$$

(13)

$$\int_1^2 \int_x^{x^2} x^2 y \, dy \, dx$$

$$= \int_1^2 \left[x^2 \cdot \frac{y^2}{2} \right]_{y=x}^{y=x^2} dx$$

$$= \int_1^2 \left[\frac{x^2 (x^2)^2}{2} - \frac{x^2 x^2}{2} \right] dx$$

$$= \int_1^2 \left(\frac{x^6}{2} - \frac{x^4}{2} \right) dx$$

$$= \left[\frac{x^7}{14} - \frac{x^5}{10} \right]_1^2$$

$$= \left(\frac{128}{14} - \frac{32}{10} \right) - \left(\frac{1}{14} - \frac{1}{10} \right)$$

$$= \frac{836}{140}$$

or $\frac{209}{35}$