① Integration by Parts

	D	I
+	$2x^3+5x+1$	e^{2x}
−	$6x^2+5$	$e^{2x}/2$
+	$12x$	$e^{2x}/4$
−	12	$e^{2x}/8$
		$e^{2x}/16$

$$\int (2x^3+5x+1)e^{2x}\,dx = e^{2x}\left[\frac{2x^3+5x+1}{2} - \frac{6x^2+5}{4} + \frac{12x}{8} - \frac{12}{16}\right] + c$$

$$= e^{2x}\left[x^3 + \frac{5}{2}x + \frac{1}{2} - \frac{6}{4}x^2 - \frac{5}{4} + \frac{12}{8}x - \frac{12}{16}\right] + c$$

$$= e^{2x}\left[x^3 - \frac{3}{2}x^2 + 4x - \frac{3}{2}\right] + c$$

(2) $\int e^{\cos 2x} \sin x \cos x \, dx$

$$\boxed{\begin{aligned} u &= \cos 2x \\ du &= -2\sin 2x\,dx \\ &= -4\sin x\cos x\,dx \\ \sin x\cos x\,dx &= \frac{-du}{4} \end{aligned}}$$

$= -\frac{1}{4}\int e^u \, du$

$= -\frac{1}{4}e^u + c$

$= -\frac{1}{4}e^{\cos 2x} + c$

(3) $\int \frac{\sec^2 x}{2+\tan x}\,dx$

$$\boxed{\begin{aligned} u &= 2+\tan x \\ du &= \sec^2 x\,dx \end{aligned}}$$

$= \int \frac{du}{u}$

$= \ln|u| + C$

$= \ln|2+\tan x| + C$

(4) $\int \frac{x^2-1}{x+3}\,dx$

Long Division

$$\begin{array}{r} x \quad -3 \\ (x+3)\;\overline{)\,x^2+0x-1} \\ -(x^2+3x) \\ \hline -3x-1 \\ -(-3x-9) \\ \hline 8 \end{array}$$

$\frac{x^2-1}{x+3} = x-3+\frac{8}{x+3}$

$\int \frac{x^2-1}{x+3}\,dx = \int \left(x-3+\frac{8}{x+3}\right)dx$

$= \frac{x^2}{2} - 3x + 8\ln|x+3| + C$

(5) $\displaystyle\int \frac{e^x}{\sqrt{3-e^{2x}}}\,dx$

$\displaystyle = \int \frac{e^x}{\sqrt{\sqrt{3}^2-(e^x)^2}}\,dx$

$$\boxed{\begin{array}{l} u=e^x \\ du=e^x\,dx \end{array}}$$

$\displaystyle = \int \frac{du}{\sqrt{\sqrt{3}^2-u^2}}$

$\displaystyle = \sin^{-1}\left(\frac{u}{\sqrt{3}}\right)+c$

$\displaystyle = \sin^{-1}\left(\frac{e^x}{\sqrt{3}}\right)+c$

(6) $\displaystyle\int \frac{1}{\sqrt{x}\,(1+3\sqrt{x})}\,dx$

$$\boxed{\begin{array}{l} u=1+3\sqrt{x} \\ du=\dfrac{3}{2\sqrt{x}}\,dx \\ \dfrac{dx}{\sqrt{x}}=\dfrac{2}{3}\,du \end{array}}$$

$\displaystyle = \frac{2}{3}\int \frac{du}{u}$

$\displaystyle = \frac{2}{3}\ln|u|+c$

$\displaystyle = \frac{2}{3}\ln|1+3\sqrt{x}|+c$

⑦ $\int_1^2 x^3 \ln x \, dx$

Integration by Parts

	D	I	
(u)	$\ln x$	x^3	(dv)
(du)	$\frac{1}{x}$	$\frac{x^4}{4}$	(v)

$$\int u\,dv = uv - \int v\,du$$

$$\int x^3 \ln x \, dx = \frac{x^4}{4} \ln x - \int \frac{x^3}{4} dx$$

$$= \frac{x^4}{4} \ln x - \frac{x^4}{16} + C$$

$$\int_1^2 x^3 \ln x \, dx = \left[\frac{x^4}{4} \ln x - \frac{x^4}{16} \right]_1^2$$

$$= (4 \ln 2 - 1) - \left(\frac{1}{4} \ln 1 - \frac{1}{16}\right)$$

$$= 4 \ln 2 - 1 + \frac{1}{16}$$

$$= 4 \ln 2 - \frac{15}{16}$$

(8)

$$\int_{\pi/6}^{\pi/3} \frac{1}{1+\cos x}\,dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{1}{1+\cos x}\cdot\frac{(1-\cos x)}{(1-\cos x)}\,dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{1-\cos x}{1-\cos^2 x}\,dx$$

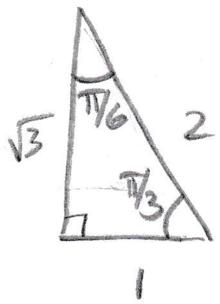
$$= \int_{\pi/6}^{\pi/3} \frac{1-\cos x}{\sin^2 x}\,dx$$

$$= \int_{\pi/6}^{\pi/3} \left(\frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x}\right)dx$$

$$= \int_{\pi/6}^{\pi/3} (\csc^2 x - \csc x\cot x)\,dx$$

$$= \Big[-\cot x + \csc x\Big]_{\pi/6}^{\pi/3}$$

$$= \left[-\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}\right] - \left[-\sqrt{3} + 2\right]$$



$$= \frac{1}{\sqrt{3}} + \sqrt{3} - 2$$

$$= \frac{\sqrt{3}}{3} + \frac{3\sqrt{3}}{3} - \frac{6}{3}$$

$$= \frac{4\sqrt{3}-6}{3}$$

(9) $\int \frac{e^{2x}}{4+e^{2x}} dx$

$$\boxed{\begin{aligned} u &= 4+e^{2x} \\ du &= 2e^{2x} dx \\ e^{2x} dx &= \frac{du}{2} \end{aligned}}$$

$= \frac{1}{2} \int \frac{du}{u}$

$= \frac{1}{2} \ln|u| + C$

$= \frac{1}{2} \ln|4+e^{2x}| + C$

(10) $\displaystyle\int_{-4}^{2} \frac{4}{x^2+2x+10}\,dx$

Complete the square

$$x^2+2x+10 = (x+1)^2 + ?$$
$$= (x+1)^2 + 9$$
$$= (x+1)^2 + 3^2$$

$$\int_{-4}^{2} \frac{4}{x^2+2x+10}\,dx = \int_{-4}^{2} \frac{4}{(x+1)^2+3^2}\,dx$$

$u = x+1$

$du = dx$

When $x=-4$ $u=-3$

$x=2$ $u=3$

$$= 4\int_{-3}^{3} \frac{du}{u^2+3^2}$$
$$= \left[\frac{4}{3}\tan^{-1}\left(\frac{u}{3}\right)\right]_{-3}^{3}$$
$$= \frac{4}{3}\tan^{-1} 1 - \frac{4}{3}\tan^{-1}(-1)$$
$$= \frac{4}{3}\left(\frac{\pi}{4}\right) - \frac{4}{3}\left(\frac{-\pi}{4}\right)$$
$$= \frac{2\pi}{3}$$

(11) $\int \cos 4x \sqrt{1+\sin 4x}\, dx$

$$\boxed{\begin{aligned} u &= 1+\sin 4x \\ du &= 4\cos 4x\, dx \\ \cos 4x\, dx &= \frac{du}{4} \end{aligned}}$$

$= \frac{1}{4} \int \sqrt{u}\, du$

$= \frac{1}{4} \left[\frac{2}{3} u^{3/2} \right] + c$

$= \frac{1}{6} u^{3/2} + c$

$= \frac{1}{6} (1+\sin 4x)^{3/2} + c$

(12) $\displaystyle\int \frac{1-\sin x}{\cos x}\,dx$

$$= \int \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right) dx$$

$$= \int (\sec x - \tan x)\,dx$$

$$= \ln|\sec x + \tan x| + \ln|\cos x| + c$$

$$= \ln|(\sec x + \tan x)\cdot \cos x| + c$$

$$= \ln|1 + \sin x| + c$$

(13) $\int \frac{dx}{x\sqrt{\ln x}}$

$u = \ln x$

$du = \frac{dx}{x}$

$= \int \frac{du}{\sqrt{u}}$

$= 2\sqrt{u} + C$

$= 2\sqrt{\ln x} + C$

(14) $\int \frac{x+1}{x^2-4}\,dx$

$$= \int \frac{x+1}{(x-2)(x+2)}\,dx$$

Partial Fractions

$$\frac{x+1}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$\boxed{x+1 = A(x+2) + B(x-2)}$$

Sub $x=2$: $\quad 3 = 4A \qquad A = \frac{3}{4}$

Sub $x=-2$: $\quad -1 = -4B \qquad B = \frac{1}{4}$

Integral $= \int \left(\frac{3}{4} \cdot \frac{1}{x-2} + \frac{1}{4} \cdot \frac{1}{x+2}\right) dx$

$$= \frac{3}{4} \ln|x-2| + \frac{1}{4} \ln|x+2| + C$$