

25.2 Cont'd

Ex: Evaluate $\int \frac{11x}{\sqrt{x^2+4}} dx$

$u = x^2 + 4$

$du = 2x dx$

$11x dx = ?$

$\rightarrow x dx = \frac{du}{2}$

$11x dx = \frac{11du}{2}$

$\int = \int \frac{11}{2} \frac{1}{\sqrt{u}} du$

$= \frac{11}{2} \int u^{-1/2} du$

$= \frac{11}{2} (2u^{1/2}) + C$

$= 11(x^2+4)^{1/2} + C$

ASIDE

$$\int x^3 dx = \frac{x^4}{4} + C$$

$$\int x^{-1/2} dx = \frac{x^{1/2}}{(1/2)} + C$$

$$= 2x^{1/2} + C$$

Ex: Evaluate $\int \frac{4z-6}{\sqrt{z^2-3z}} dz$

let $u = z^2 - 3z$

$du = (2z - 3) dz$

$$(4z-6)dz = ?$$

$$(4z-6)dz = 2du$$

$$\int = \int \frac{2du}{\sqrt{u}}$$

$$= 2 \int u^{-1/2} du$$

$$= 2(2u^{1/2}) + C$$

$$= 4(z^2 - 3z)^{1/2} + C$$

Ex: Evaluate

a) $\int (2+3x^2)^2 dx$

Not substitution

expand

$$= \int (4 + 12x^2 + 9x^4) dx$$

$$= 4x + \frac{12x^3}{3} + \frac{9x^5}{5} + C$$

$4x^3$

b) $\int \frac{x^3 + x^2}{\sqrt{x}} dx$

simplify

Not substitution (can't rewrite with

variable u)

$$\begin{aligned} &= \int x^{-1/2} (x^3 + x^2) dx \\ &= \int (x^{5/2} + x^{3/2}) dx \\ &= \frac{2}{7} x^{7/2} + \frac{2}{5} x^{5/2} + C \end{aligned}$$

Ex: Find y given $\frac{dy}{dx} = 7x^2$
and $(1, 3)$ lies on the curve y .

$$\frac{dy}{dx} = 7x^2$$

Integrate:

$$y = \frac{7x^3}{3} + C$$

Formally:

$$\begin{cases} dy = 7x^2 dx \\ \int dy = \int 7x^2 dx \\ y = \frac{7x^3}{3} + C \end{cases}$$

$$\begin{matrix} y=3 \\ x=1 \end{matrix} :$$

$$3 = \frac{7}{3} + C$$

$$C = \frac{2}{3}$$

$$\frac{9}{3} = \frac{7}{3} + C$$

$$y = \frac{7x^3}{3} + \frac{2}{3}$$

Ex: $f''(x) = 3x + 1$
Find $f(x)$ given $f(0) = 2$ and $f(1) = 4$

$$\begin{aligned}
 f'(x) &= \int f''(x) dx \\
 &= \int (3x+1) dx \\
 &= \frac{3x^2}{2} + x + C
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= \int f'(x) dx \\
 &= \int \left(\frac{3x^2}{2} + x + C \right) dx
 \end{aligned}$$

$$f = \frac{x^3}{2} + \frac{x^2}{2} + Cx + D$$

constant
(not C)

$$\begin{aligned}
 f=2 \\
 x=0 : \quad 2 = 0 + 0 + 0 + D \\
 \quad \quad \quad D=2
 \end{aligned}$$

$$f = \frac{x^3}{2} + \frac{x^2}{2} + Cx + 2$$

$$\begin{aligned}
 f=4 \\
 x=1 : \quad 4 = \frac{1}{2} + \frac{1}{2} + C + 2 \\
 \quad \quad \quad C=1
 \end{aligned}$$

$$f = \frac{x^3}{2} + \frac{x^2}{2} + x + 2$$