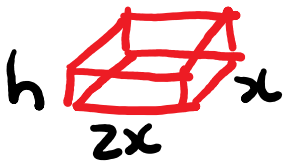
Week 8 Thursday

October 25, 2018 9:21 AM

Review

① Rectangular box
Length = 2·width
3 Dimensions sum to 36cm
Find the max. volume

1)



Let width = x

Constraint

$$x+2x+h=36$$

$$\text{Max } V=2x^2h$$

2) Single variable

Constraint $h=36-3x \rightarrow V$

$$V=2x^2(36-3x)$$

3) C.P.

$$V=72x^2-6x^3$$

$$V'=144x-18x^2$$

$$144x - 18x^2 = 0$$

$$18x(8 - x) = 0$$

$x = 0$ $\quad$ $x = 8$

4) Answer

$$V = 2(8^2)(36 - 24)$$

$$= 1536 \text{ cm}^3$$

(2) An astronaut weighs

$$w(h) = 650\left(1 + \frac{h}{6400}\right)^{-1} \text{ N at } h \text{ km}$$

above sea level.

Space shuttle is climbing at 10 km/s.

Find $\frac{dw}{dt}$ when $h = 2000$ km.

$\frac{dh}{dt} = 10$ $\quad$ $\frac{dw}{dt} = ?$ $\quad$ $h = 2000$ km

$$\frac{dw}{dt} = \frac{dw}{dh} \cdot \frac{dh}{dt}$$

$$= \left[-650\left(1 + \frac{h}{6400}\right)^{-2} \cdot \frac{1}{6400}\right] \frac{dh}{dt}$$

$$= \frac{-650}{6400}\left(1+\frac{2000}{6400}\right)^{-2}\cdot 10$$

$$\approx -0.59 \quad N/s$$

③ $V = \frac{4}{3}\pi r^3$

Show that $\frac{dV}{V} = 3\frac{dr}{r}$

Calculate $\frac{dV}{V} = \frac{4\pi r^2 dr}{\left(\frac{4}{3}\pi r^3\right)}$

$$= \frac{3}{4}\cdot\frac{4\pi r^2 dr}{\pi r^3 r}$$

$$= 3\frac{dr}{r}$$

④ Find all points of inflection

$$f = \frac{x^5}{20} + \frac{x^4}{12}$$

~~$f' = \frac{5x^4}{20} + \frac{4x^3}{12}$~~ $f' = \frac{x^4}{4} + \frac{x^3}{3}$

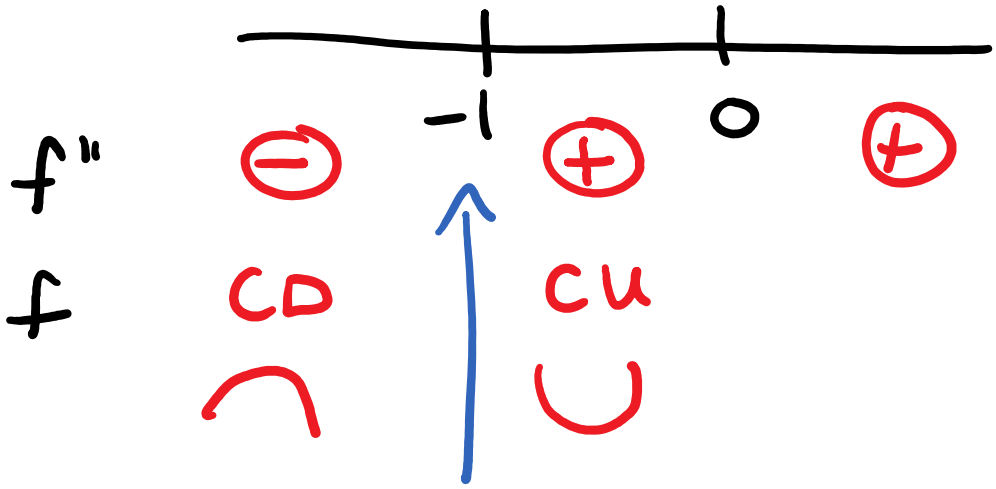
$f'' = x^3 + x^2$

P. of I. Set $f'' = 0$

$$x^3 + x^2 = 0$$

$$x^2(x+1) = 0 \leftarrow$$

$$x = 0, -1$$



Point of inflection $(-1, \quad)$

$f(-1)$

$$= \frac{(-1)^5}{20} + \frac{(-1)^4}{12}$$

$$= \frac{-1}{20} + \frac{1}{12}$$

$$= \frac{8}{240} \text{ or } \frac{1}{30}$$