Week 7 Wednesday

October 17, 2018 11:09 AM

Quiz Wed 24.8

27.3 Derivatives of Inverse Trig Functions

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}} \quad *$$

$$\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2} \quad *$$

Ex: Find $f'(x)$

a) $f(x) = \sin^{-1} 7x$

$$f'(x) = \frac{1}{\sqrt{1-(7x)^2}} \cdot 7$$

$$= \frac{7}{\sqrt{1-49x^2}}$$

b) $f(x) = \frac{1}{3}\sin^{-1}\left(\frac{x}{4}\right)$

$$f'(x) = \frac{1}{3}\cdot\frac{1}{\sqrt{1-\left(\frac{x}{4}\right)^2}}\cdot\frac{1}{4}$$ ✓

$$= \frac{1}{3\sqrt{1-\frac{x^2}{16}}}\cdot\frac{1}{\sqrt{16}}$$

$$= \frac{1}{3\sqrt{16-x^2}}$$ ✓

c) $f(x) = (x)(\cos^{-1}x^2)$

$$f'(x) = x\underbrace{\frac{d}{dx}\cos^{-1}x^2} + \cos^{-1}x^2\,(1)$$

$$= x\left[\frac{-1}{\sqrt{1-(x^2)^2}}\cdot 2x\right] + \cos^{-1}x^2$$

$$= \frac{-2x^2}{\sqrt{1-x^4}} + \cos^{-1}x^2$$

d) $f(x) = [\tan^{-1}(8x) - 5x^2]^3$

$$f'(x) = 3\left[\tan^{-1}8x - 5x^2\right]^2\left[\frac{d}{dx}\tan^{-1}8x - 10x\right]$$

$$= 3\left[\tan^{-1}8x - 5x^2\right]^2\left[\frac{1}{1+(8x)^2}\cdot 8 - 10x\right]$$

$$= 3\left[\tan^{-1}8x - 5x^2\right]^2\left[\frac{8}{1+64x^2} - 10x\right]$$ ✓

e) $f(x) = \tan^{-1}\frac{k}{x}$ k: constant

$$f'(x) = \frac{1}{1+\left(\frac{k}{x}\right)^2}\cdot\frac{d}{dx}\left[kx^{-1}\right]$$

$$= \frac{1}{\left(1+\frac{k^2}{x^2}\right)}\left(-\frac{k}{x^2}\right)$$ ✓ ($-kx^{-2}$)

or $\frac{-k}{x^2+k^2}$ ✓