Week 6 Thursday

October 11, 2018 1:18 PM

24.8 Cont'd

ERROR

Absolute error: true − measured

Relative error: $\frac{\text{true} - \text{measured}}{\text{measured}}$

(Sometimes R.E. $= \frac{M-T}{T}$)

Notation:

measured value $= x$

true " $= x + dx$

absolute error $= dx$

relative error $= \frac{dx}{x}$

Ex: Measured distance $= 3.2\,m \leftarrow x$

True distance $= 3.25\,m \leftarrow x + dx$

Find absolute and relative error.

absolute error $dx = 0.05\,m$

relative error $\frac{dx}{x} = \frac{0.05\,m}{3.2\,m}$

≈ 0.0156

or 1.56%

$$\boxed{\text{Recall if } y = f(x) \quad dy = f'(x)\,dx}$$

<u>Ex</u>: Given $A = \pi r^2$ and $\frac{dr}{r} = 0.02$, find $\frac{dA}{A}$

$$dA = 2\pi r\,dr$$

$$\frac{dA}{A} = \frac{2\cancel{\pi}\cancel{r}\,dr}{\cancel{\pi}\cancel{r^2}\,r}$$

$$= \frac{2\,dr}{r}$$

$$= 2\left(\frac{dr}{r}\right)$$

$$= 2(0.02)$$

$$= 0.04$$

<u>Interpretation</u>:

2% relative error in radius leads to a 4% R.E. in circle's area.

Why does the R.E. double?

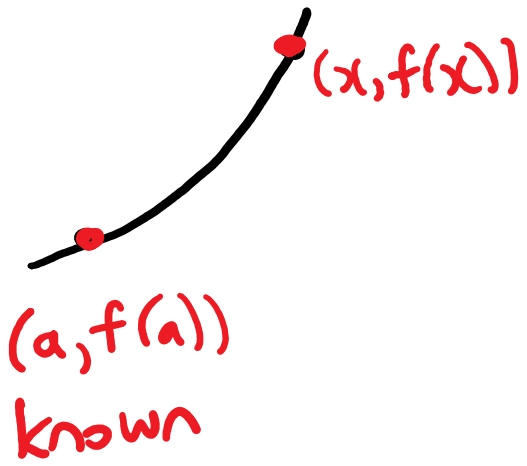
$r^{\boxed{2}}$ Dimension = 2

Ex: Consider a sphere

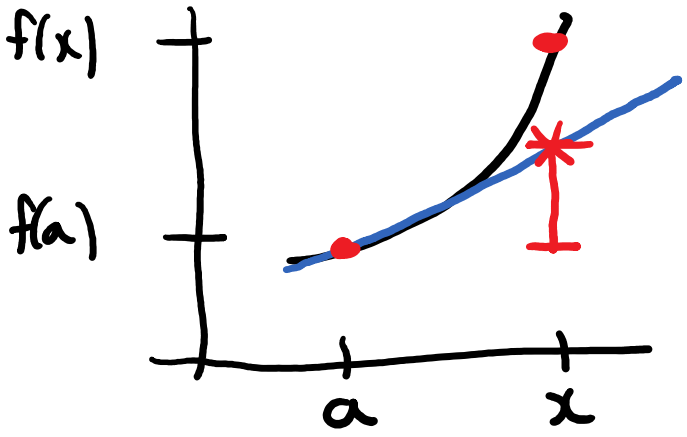
$$\frac{dV}{V} = ? \frac{dr}{r}$$

$$\frac{dV}{V} = 3\frac{dr}{r}$$

Linear Approximation



Goal: Estimate $f(x)$



$$f(x) \approx f(a) + \text{rise}$$

$$\boxed{m_{tan} = \frac{\text{rise}}{\text{run}}}$$

$$\approx f(a) + m_{tan} \cdot run$$

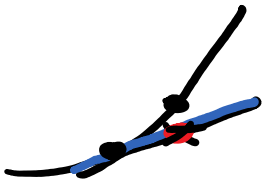
$$\approx f(a) + f'(a) \cdot (x-a)$$

on formula sheet

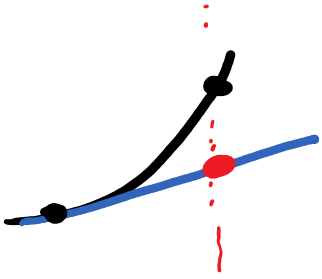
$$\boxed{f(x) \approx f(a) + f'(a) \cdot (x-a)}$$

Linear approximation of $f(x)$ at $x=a$

Linearization "

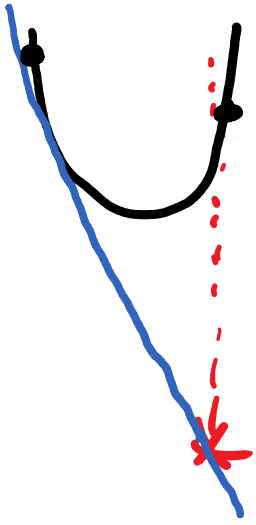


Better Approx.



Worse

As $x \to a$, error $\to 0$



Much worse

Ex: a) Find the linearization

of $f(x) = x^4$ at $x = 2$

$\boxed{a = 2}$ Constant

$f(2) = 16$

$f'(x) = 4x^3$

$f'(2) = 32$

$f(x) \approx f(a) + f'(a)(x-a)$

$f(x) \approx 16 + 32(x-2)$

b) Estimate 1.98^4

Let $x = 1.98$

(Better approx. the closer x is to 2)

$f(1.98) \approx 16 + 32(1.98-2)$

≈ 15.36

You might see

$(1+x)^N \approx 1 + Nx$ fluids physics

$\sin\theta \approx \theta$ physics surveying?