Week 6 Wednesday

October 10, 2018 1:05 PM

Class observation tomorrow

Quiz Wed 17th 24.7

24.8 Differentials and Linear Approximation

Application of differentials:
absolute or relative error

Ex: $y = 3x^6 + 7x^3$

$$\frac{dy}{dx} = 18x^5 + 21x^2$$

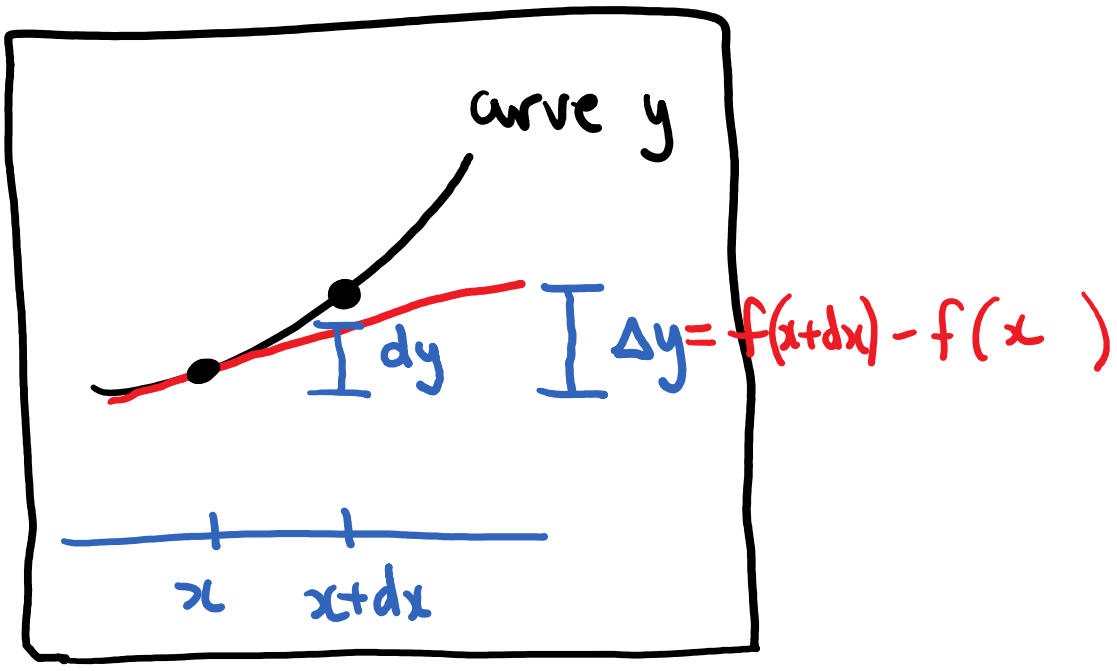
$$dy = (18x^5 + 21x^2)\,dx$$

"differential of y" "differential of x"

If $y = f(x)$
then $dy = f'(x)\,dx$

Ex: $V = \frac{4}{3}\pi r^3$

$$dV = 4\pi r^2\,dr$$



Δx or dx : small change in x

dy : rise of tangent line

Δy : true change in y

When dx is small, $\Delta y \approx dy$

Δy: hard to calculate

dy: easy

Ex: $f(x) = x^2 + 2$

Compute Δy and dy given $x = 1$ and $dx = 0.05$

$$dy = f'(x)\,dx$$
$$\Delta y = f(x+dx) - f(x)$$

approx change in y

$dy = 2x\,dx$
$= 2(1)(0.05)$
$= 0.1$

easy

$\Delta y = f(1.05) - f(1)$
$= (1.05^2 + 2) - 3$
$= 0.1025$

harder

true change in y

Ex: Sphere's radius is measured as 2.500 cm. If the true radius is 2.512 cm, estimate the error in the volume.

$r = 2.500$ measured

$dr =$ true − measured
$= 2.512 - 2.500$
$= 0.012$ error

$V = \frac{4}{3}\pi r^3$

$\Delta V \approx dV$

ΔV ↑ true error

$= 4\pi r^2 dr$

$= 4\pi (2.500)^2 (0.012)$

$\approx 0.9425 \text{ cm}^3$