Week 14 Friday

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Full solutions to review problems
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(15) a) $\int 2x^2\sqrt{1-4x^3}\,dx$

$u = 1-4x^3$
$du = -12x^2\,dx$
$-12x^2\,dx = du$
$x^2\,dx = -\frac{1}{12}\,du$

$= -\frac{2}{12}\int \sqrt{u}\,du$

$= -\frac{1}{6}\int u^{1/2}\,du$

$= -\frac{1}{6}\left[\frac{2}{3}u^{3/2}\right] + C$

$= -\frac{1}{9}\left(1-4x^3\right)^{3/2} + C$

b) $\int \frac{2x-1}{x^3}\,dx$ ← simplify

$= \int x^{-3}(2x-1)\,dx$

$= \int (2x^{-2} - x^{-3})\,dx$

$= 2(-x^{-1}) - \left(-\frac{1}{2}x^{-2}\right) + C$

$= -2x^{-1} + \frac{1}{2}x^{-2} + C$

(21)

Arc Length

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$y = 0.04x^{3/2}$

$\frac{dy}{dx} = 0.06x^{1/2}$

$\left(\frac{dy}{dx}\right)^2 = 0.0036x$

$0 \le x \le 100$

$$s = \int_0^{100} \sqrt{1 + 0.0036x}\,dx$$

$u = 1 + 0.0036x$
$du = 0.0036\,dx$

$du = 0.0036\,dx$

$dx = \dfrac{du}{0.0036}$

ALTERNATIVELY

$x=0 \rightarrow u=1$

$x=100 \rightarrow u=1.36$

$$\text{Answer} = \frac{1}{0.0036} \times \frac{2}{3}\left[u^{3/2}\right]_1^{1.36}$$

$$\approx 108.52\text{ m}$$

$$= \frac{1}{0.0036}\int_{x=0}^{x=100} \sqrt{u}\,du$$

$$= \frac{1}{0.0036}\left[\frac{2}{3}u^{3/2}\right]_{x=0}^{x=100}$$

$$= \frac{2}{3(0.0036)}\left[(1+0.0036x)^{3/2}\right]_0^{100}$$

$$= \frac{2}{3(0.0036)}\left[1.36^{3/2} - 1\right]$$

$$\approx 108.52\text{ m}$$

(17)

$$a = 12t$$

$$v = \int 12t\,dt$$

$$v = 6t^2 + C_1$$

$v=5$, $t=0$: $C_1 = 5$

$$\boxed{v = 6t^2 + 5}$$

$$s = \int (6t^2+5)\,dt$$

$$s = \frac{6t^3}{3} + 5t + C_2$$

$s=0$, $t=0$: $C_2 = 0$

$$\boxed{s = 2t^3 + 5t}$$

(22) x y z #

(23)

$$\begin{array}{cccc} x & y & z & \# \\ \left[\begin{array}{ccc|c} 2 & 8 & -10 & -2 \\ 3 & 5 & 6 & 4 \\ 4 & 2 & 22 & 10 \end{array}\right] \end{array}$$

$\frac{R_1}{2}$

$$\left[\begin{array}{ccc|c} 1 & 4 & -5 & -1 \\ 3 & 5 & 6 & 4 \\ 4 & 2 & 22 & 10 \end{array}\right]$$

$R_2 - 3R_1$, $R_3 - 4R_1$

$$\left[\begin{array}{ccc|c} 1 & 4 & -5 & -1 \\ 0 & -7 & 21 & 7 \\ 0 & -14 & 42 & 14 \end{array}\right]$$

$\frac{R_2}{-7}$

$$\left[\begin{array}{ccc|c} 1 & 4 & -5 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & -14 & 42 & 14 \end{array}\right]$$

$$\begin{array}{cccc} x & y & z & \# \\ \left[\begin{array}{ccc|c} 1 & 0 & 7 & 3 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$R_1 - 4R_2$

$R_3 + 14R_2$

RREF

$x + 7z = 3 \rightarrow$

$y - 3z = -1 \rightarrow$

z = anything	$z = t$
$x = 3 - 7z$	$x = 3 - 7t$
$y = -1 + 3z$	$y = -1 + 3t$

(22) a) $[A \mid I]$

$\rightsquigarrow [I \mid A^{-1}]$

1) $V - A^{-1}R$

b) $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} = \\ = \\ = \end{bmatrix} \begin{bmatrix} -11 \\ -14 \\ 9 \end{bmatrix}$$

$$= \begin{bmatrix} \# \\ \# \\ \# \end{bmatrix}$$