

# Quiz Wed 16.2

## 16.5 Gauss-Jordan Elimination Cont'd

Systems with no solution  
 ∆-many solutions

Ex: 
$$\begin{array}{c|ccc} x & y & z & \# \\ \hline 1 & 2 & 1 & 9 \\ 1 & 3 & 3 & 12 \\ 1 & 4 & 5 & 1 \end{array}$$

$R_2 - R_1$   
 $R_3 - R_1$  
$$\begin{array}{c|ccc} 1 & 2 & 1 & 9 \\ \hline 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & -8 \end{array}$$

$R_1 - 2R_2$   
 $R_3 - 2R_2$  
$$\begin{array}{c|ccc} x & y & z & \# \\ \hline & & & \\ 0 & 0 & 0 & -14 \end{array}$$

$0x + 0y + 0z = -14$   
 impossible

System has no solution  
 "System is inconsistent"

If you see a row like this  
then the system has no solution :

$$[0 \ 0 \ 1 \ 3]$$

$$[0 \ 0 \ 0 \ -11]$$

$$[\text{all zeros} \mid \text{nonzero}]$$

Ex:

$$\begin{array}{c|ccc} x & y & z & \# \\ \hline 1 & 2 & 1 & 9 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 6 \end{array}$$

$$R_1 - 2R_2 \quad \begin{array}{c|ccc} 1 & 0 & -3 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}$$

$$R_3 - 2R_2 \quad \begin{array}{c|ccc} 1 & 0 & -3 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}$$

RREF ✓

$$\begin{array}{c|ccc} x & y & z & \# \\ \hline 1 & 0 & -3 & 3 \\ 0 & 1 & 2 & 3 \end{array}$$

↑

Any column without a  
leading 1 gets a parameter

$$z = t \quad \text{any real } \#$$

$$1x + 0y - 3z = 3$$

$$x = 3 + 3z$$

$$x = 3 + 3t$$

$$\begin{aligned} y + 2z &= 3 \\ y &= 3 - 2z \\ y &= 3 - 2t \end{aligned}$$

$$(x, y, z) = (3 + 3t, 3 - 2t, t)$$

$\infty$ -many solutions

ASIDE Some particular solutions:

$$t = 2 \quad (x, y, z) = (9, -1, 2) \quad \checkmark$$

$$t = 0 \quad (x, y, z) = (3, 3, 0) \quad \checkmark$$

$$t = 17 \quad (x, y, z) = (54, -31, 17) \quad \checkmark$$

Not all triples are solutions

A non-solution :  $(1, 1, 1)$

Ex: Solve

$$\begin{array}{cccc|c} x & y & z & \# & \\ \hline 4 & -8 & 12 & 20 & \\ 3 & 1 & 1 & 5 & \end{array}$$

$$R_1 / 4 \quad \begin{array}{ccc|c} 1 & -2 & 3 & 5 \\ \hline 3 & 1 & 1 & 5 \end{array}$$

$$R_1 \rightarrow \begin{bmatrix} 1 & -2 & 3 & | & 5 \\ 0 & 7 & -8 & | & -10 \end{bmatrix}$$

$$R_2 - 3R_1 \rightarrow \begin{bmatrix} 1 & -2 & 3 & | & 5 \\ 0 & 7 & -8 & | & -10 \end{bmatrix}$$

$$R_2/7 \rightarrow \begin{bmatrix} 1 & -2 & 3 & | & 5 \\ 0 & 1 & -\frac{8}{7} & | & -\frac{10}{7} \end{bmatrix}$$

$$R_1 + 2R_2 \rightarrow \begin{bmatrix} 1 & 0 & \frac{5}{7} & | & \frac{15}{7} \\ 0 & 1 & -\frac{8}{7} & | & -\frac{10}{7} \end{bmatrix}$$

$3 + 2\left(-\frac{8}{7}\right) = \frac{21}{7} - \frac{16}{7}$   
 $5 + 2\left(-\frac{10}{7}\right) = \frac{35}{7} - \frac{20}{7}$

RREF ✓

$$\begin{bmatrix} 1 & 0 & \frac{5}{7} & | & \frac{15}{7} \\ 0 & 1 & -\frac{8}{7} & | & -\frac{10}{7} \end{bmatrix}$$

$$z = t$$

$$x + \frac{5}{7}z = \frac{15}{7}$$

$$x = \frac{15}{7} - \frac{5}{7}z$$

$$x = \frac{15}{7} - \frac{5}{7}t$$

$$y - \frac{8}{7}z = -\frac{10}{7}$$

$$y = -\frac{10}{7} + \frac{8}{7}t$$

$$(x, y, z) = \left( \frac{15}{7} - \frac{5}{7}t, -\frac{10}{7} + \frac{8}{7}t, t \right)$$