Week 13 Thursday

November 29, 2018 9:14 AM

16.4 Cont'd

Ex: Solve by finding A^{-1}

$$\begin{cases} x + 2y + 2z = 17 \\ x + y + z = 9 \\ x - 2z = -9 \end{cases}$$

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}$$

$$[A|I] \rightsquigarrow [I|A^{-1}]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -2 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - R_1$, $R_3 - R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & -2 & -4 & -1 & 0 & 1 \end{array}\right]$$

current row $\pm$ # (pivot row)

Get a 1

$R_2/(-1)$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & -2 & -4 & -1 & 0 & 1 \end{array}\right]$$

$R_1 - 2R_2$

$R_3 + 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & -2 & 1 & -2 & 1 \end{array}\right]$$

Get a 1

$$R_3/(-2)\quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 1 & -\frac{1}{2} \end{array}\right]$$

$$R_2-R_3\quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & \frac{3}{2} & -2 & \frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & 1 & -\frac{1}{2} \end{array}\right] \leftarrow A^{-1}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} -1 & 2 & 0 \\ \frac{3}{2} & -2 & \frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 17 \\ 9 \\ -9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

16.5 Gauss-Jordan Elimination

$$\begin{cases} x+2y=7 \\ 3x+4y=15 \end{cases}$$

will be written:

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ 3 & 4 & 15 \end{array}\right]$$

A matrix is in **Reduced Row Echelon Form** (RREF) if:

i) The first nonzero entry in each ROW is a 1;

ii) These leading 1's have 0's everywhere else in their COLUMNS;

iii) These leading 1's move down and to the right; and

iv) Any zero rows are located at the bottom of the matrix.

Ex:

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ RREF

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ RREF

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ not RREF

$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ not RREF

Ex: Solve using Gauss-Jordan Elimination

$$\begin{cases} 3x - 4y + z = 25 \\ 2x + 4y + z = -16 \\ x + 5z = 11 \end{cases}$$

$$\begin{matrix} x & y & z & \# \end{matrix}$$
$$\left[\begin{array}{ccc|c} 3 & -4 & 1 & 25 \\ 2 & 4 & 1 & -16 \\ 1 & 0 & 5 & 11 \end{array}\right]$$

Left side → RREF
using our 3 types of row operations

$R_1 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 11 \\ 2 & 4 & 1 & -16 \\ 3 & -4 & 1 & 25 \end{array}\right]$$

$R_2 - 2R_1$, $R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 11 \\ 0 & 4 & -9 & -38 \\ 0 & -4 & -14 & -8 \end{array}\right]$$

$R_2/4$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 11 \\ 0 & 1 & -9/4 & -38/4 \\ 0 & -4 & -14 & -8 \end{array}\right]$$

$R_3 + 4R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 11 \\ 0 & 1 & -9/4 & -38/4 \\ 0 & 0 & -23 & -46 \end{array}\right]$$

$R_3/(-23)$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 11 \\ 0 & 1 & -9/4 & -38/4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_1 - 5R_3$, $R_2 + \frac{9}{4}R_3$

$$\begin{array}{ccc|c} x & y & z & \# \\ \hline 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{array} \leftarrow -\frac{38}{4} + \frac{9}{4}(2)$$

RREF

$1x + 0y + 0z = 1 \rightarrow$

$x = 1$

$y = -5$

$z = 2$