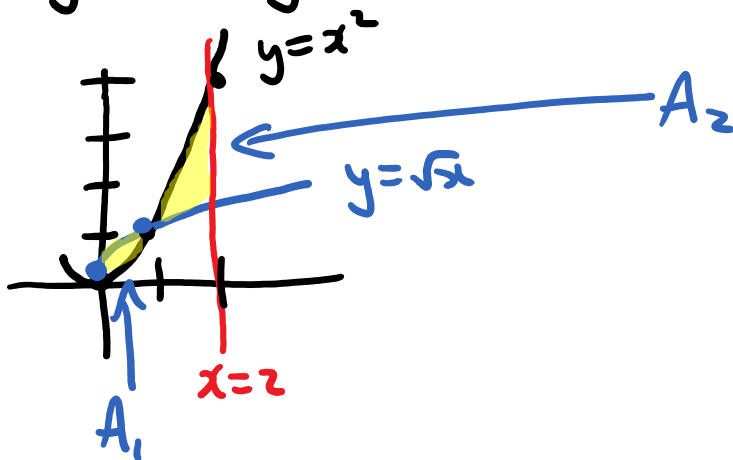


# Quiz Wed 21<sup>st</sup> 26.2

## 26.2 Area Cont'd

Ex: Total area bounded by  $y=x^2$ ,  $y=\sqrt{x}$ ,  $x=0$  and  $x=2$ ?



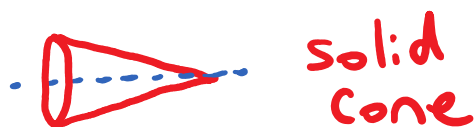
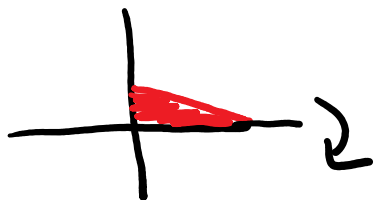
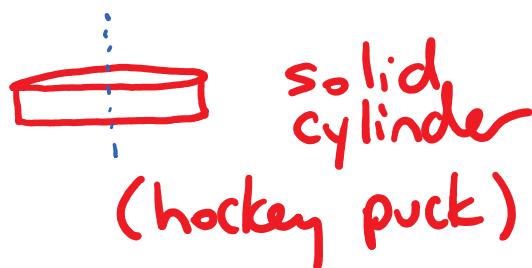
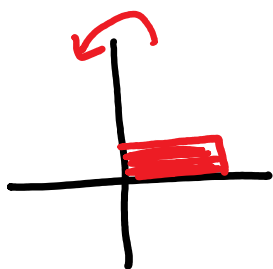
$$\begin{aligned}
 A_1 &= \int_0^1 (y_{\text{top}} - y_{\text{bot}}) dx \\
 &= \int_0^1 (\sqrt{x} - x^2) dx \\
 &= \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 A_2 &= \int_1^2 (x^2 - \sqrt{x}) dx \\
 &= \left[ \frac{x^3}{3} - \frac{2}{3} x^{3/2} \right]_1^2 \\
 &= \frac{8}{3} - \frac{2}{3} (2)^{3/2} - \left[ \frac{-1}{3} \right] \\
 &= 3 - \frac{2}{3} (2)^{3/2}
 \end{aligned}$$

Total area  $A = A_1 + A_2$   
 $= \frac{10}{3} - \frac{2}{3}(2)^{3/2}$

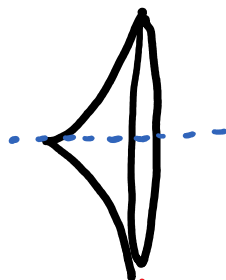
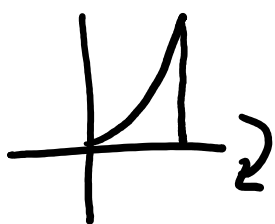
## 26.3 Volumes by Integration

Revolve an area around an axis



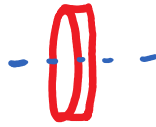
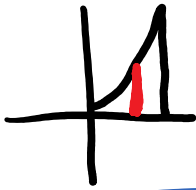
Goal: calculate volumes

### Disk Method



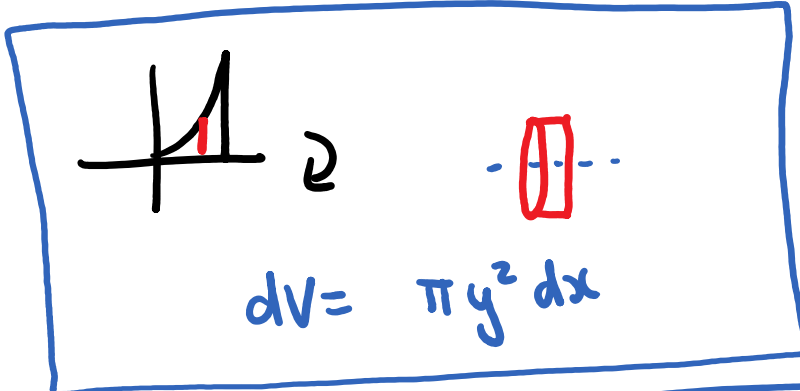
Take slices perpendicular to axis of revolution.

Produces a solid disk (hockey puck)

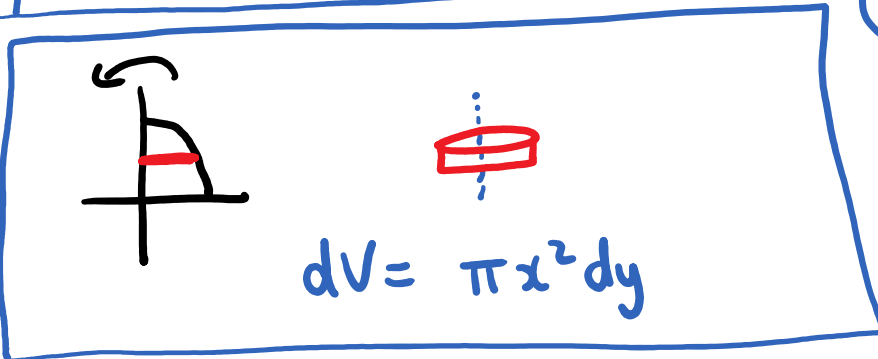


Volume of disk

$$dV = \pi \cdot \text{radius}^2 \cdot \text{thickness}$$

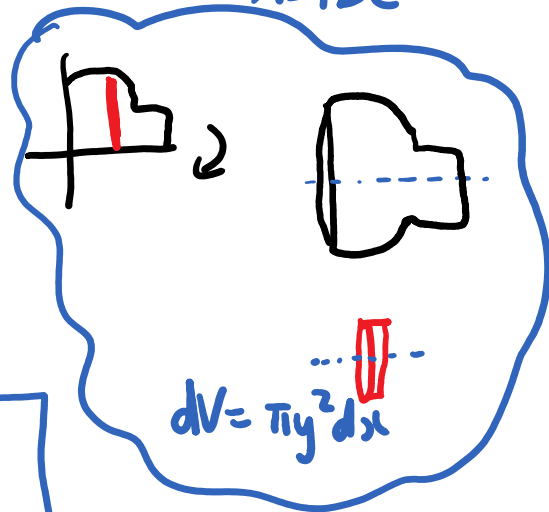


$$dV = \pi y^2 dx$$



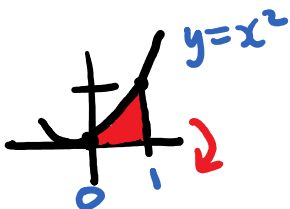
$$dV = \pi x^2 dy$$

ASIDE



$$dV = \pi y^2 dx$$

Ex: Revolve region bounded by  $y = x^2$ ,  $y = 0$ ,  $x = 1$  about  $x$ -axis.  
Volume?



disk

$$dV = \pi \cdot \text{radius}^2 \cdot \text{thickness}$$

$$dV = \pi y^2 dx$$

$$\begin{aligned} V &= \int dV \\ &= \int_0^1 \pi y^2 dx \\ &= \pi \int_0^1 (x^2)^2 dx \quad \leftarrow y^2 \\ &= \pi \int_0^1 x^4 dx \\ &= \pi \left[ \frac{x^5}{5} \right]_0^1 \\ &= \pi \left( \frac{1}{5} \right) \\ &= \frac{\pi}{5} \end{aligned}$$