

Tues 13th Class Survey 1:30 - 1:45
 Wed 14th Quiz 25.4

Ex: Ball is thrown straight up from 12m high. Takes 6s to land. Find initial velocity v_0 .

$$a(t) = -9.8 \text{ (gravity)}$$

$$v(t) = \int -9.8 dt$$

$$v(t) = -9.8t + C_1$$

$$v = v_0 : \quad v_0 = 0 + C_1 \quad \uparrow$$

$$t = 0 : \quad C_1 = v_0$$

$$v(t) = -9.8t + v_0$$

$$h(t) = \int v(t) dt$$

$$h(t) = \int (-9.8t + v_0) dt$$

$$h(t) = -\frac{9.8t^2}{2} + v_0 t + C_2$$

$$t=0 : \quad h=12 : \quad 12 = 0 + 0 + C_2 \quad \uparrow$$

$$C_2 = 12$$

$$h(t) = -4.9t^2 + v_0 t + 12$$

$$t=6 : \quad h=0 : \quad 0 = -4.9(6)^2 + v_0(6) + 12$$

$$4.9(6)^2 - 12 = 6v_0$$

$$v_0 = \frac{4.9(6)^2 - 12}{6} \approx 27.4 \text{ m/s}$$

6

Ex: Car travels in a straight line with $a = -4t$. Brakes are applied when velocity = 20 m/s. Stopping distance?

1) Find $v(t)$, $s(t)$

$$a = -4t$$

$$v = \int (-4t) dt$$

$$v = -2t^2 + C_1$$

$$\begin{matrix} v=20 \\ t=0 \end{matrix} : C_1 = 20 \rightarrow$$

$$v = -2t^2 + 20$$

$$s = \int v dt$$

$$s = \int (-2t^2 + 20) dt$$

$$s = -\frac{2t^3}{3} + 20t + C_2$$

$$\begin{matrix} s=0 \\ t=0 \end{matrix} : C_2 = 0 \rightarrow$$

$$s = -\frac{2t^3}{3} + 20t$$

2) Time to stop?

$$\text{Set } v = 0$$

$$-2t^2 + 20 = 0$$

$$\begin{matrix} t=0 \\ \text{brakes applied} \\ v(0) = 20 \text{ m/s} \\ s(0) = 0 \end{matrix}$$

$$20 = 2t^2$$

$$10 = t^2$$

$$t = \pm \sqrt{10}$$

$$t = \sqrt{10}$$

3) Stopping distance?

$$S(\sqrt{10}) = \frac{-2\sqrt{10}^3}{3} + 20\sqrt{10}$$

$$\approx 42 \text{ m}$$

Ex: Angular velocity of a rotor is $\frac{d\theta}{dt} = (4t+1)^{3/2}$
Find angular displacement θ after 5 seconds

$$\begin{aligned}\theta &= \int \boxed{\frac{d\theta}{dt}} dt \\ &= \int (4t+1)^{3/2} dt\end{aligned}$$

$$\begin{aligned}u &= 4t+1 \\ du &= 4dt \\ dt &= ? \\ dt &= \frac{du}{4}\end{aligned}$$

$$= \frac{1}{4} \int u^{3/2} du$$

$$= \frac{1}{4} \left(\frac{2}{5} u^{5/2} \right) + C$$

$$\theta = \frac{1}{10} (4t+1)^{5/2} + C$$

$$\theta = 0$$

$$t = 0$$

$$: \quad 0 = \frac{1}{10} + C$$

$$C = -\frac{1}{10}$$

$$\theta = \frac{1}{10} (4t+1)^{5/2} - \frac{1}{10}$$

$$t = 5: \quad \theta = \frac{1}{10} (21)^{5/2} - \frac{1}{10}$$

$$\approx 202 \text{ rad}$$

$$(\div 2\pi)$$

$$\approx 32 \text{ full rotations}$$