Week 10 Tuesday

November 6, 2018 8:48 AM

25.6 Cont'd

Ex: Approximate $\int_1^5 \sqrt{1+x^2}\,dx$ using Simpson's Rule with 4 intervals. Answer to 2 decimal places.

$n=4$ even ✓

$\frac{b-a}{n} = \frac{5-1}{4} = 1$

x	$y=\sqrt{1+x^2}$
1	$\sqrt{2}$
2	$\sqrt{5}$
3	$\sqrt{10}$
4	$\sqrt{17}$
5	$\sqrt{26}$

Start at a
End at b
Go up by $\frac{b-a}{n}$

1-4-2-4-1

$$\int_1^5 \sqrt{1+x^2}\,dx \approx \frac{b-a}{3n}\left[y_0+4y_1+2y_2+4y_3+y_4\right]$$

$$\approx \frac{1}{3}\left[\sqrt{2}+4\sqrt{5}+2\sqrt{10}+4\sqrt{17}+\sqrt{26}\right]$$

$$\approx 12.76$$

Ex: Approximate to 2 d.p.

Evaluate $\int_0^2 7^x\,dx$ using:

a) Trapezoidal with $n=4$

b) Simpson's "

a) $\frac{b-a}{n} = \frac{2-0}{4} = 0.5$

x	$y = 7^x$
0	1
0.5	$7^{0.5}$
1	7
1.5	$7^{1.5}$
2	49

1-2-2-2-1

$$\text{Integral} \approx \frac{b-a}{2n}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right]$$

$$\approx \frac{0.5}{2}\left[1 + 2(7^{0.5}) + 2(7) + 2(7^{1.5}) + 49\right]$$

$$\approx 26.58$$

b) $n = 4$ is even ✓

1-4-2-4-1

$$\text{Integral} \approx \frac{b-a}{3n}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$\approx \frac{0.5}{3}\left[1 + 4(7^{0.5}) + 2(7) + 4(7^{1.5}) + 49\right]$$

$$\approx 24.78$$

For reference, true value ≈ 24.67

Simpson's Rule is generally better

26.1 Applications of Integration

Recall $s(t)$ or $h(t)$ = displacement

$v(t)$ = velocity

$a(t)$ = acceleration

take derivative ↓ integrate ↑

$$v(t) = s'(t)$$
$$a(t) = v'(t) = s''(t)$$

$$v(t) = \int a(t)\,dt$$
$$a(t) = \int v(t)\,dt$$

Ex: A ball is thrown straight up from the ground with initial velocity 3 m/s. Find $h(t)$.

$a(t) = -9.8$ (gravity)

$v(t) = \int -9.8\,dt$

$v(t) = -9.8t + C_1$

$v=3, t=0$: $3 = 0 + C_1$, $C_1 = 3$

$v(t) = -9.8t + 3$

$h(t) = \int (-9.8t + 3)\,dt$

$h(t) = -\frac{9.8t^2}{2} + 3t + C_2$ (C_2: new name)

$h=0, t=0$: $0 = 0 + 0 + C_2$, $C_2 = 0$

$h(t) = -4.9t^2 + 3t$

Ex: Ball is thrown straight up from 12m high. Takes 6s to land. Find initial velocity v_0.

$a(t) = -9.8$ (gravity)

$v(t) = \int -9.8\,dt$

$v(t) = -9.8t + C_1$

$v = v_0$:
$t = 0$

$v_0 = 0 + C_1$
$C_1 = v_0$

$$\boxed{v(t) = -9.8t + v_0}$$