26.1 Applications of Integration

Recall $s(t)$ or $h(t)$ = displacement

$v(t)$ = velocity

$a(t)$ = acceleration

take derivative ↓ ↑ integrate

$$\boxed{\begin{aligned} v(t) &= s'(t) \\ a(t) &= v'(t) = s''(t) \end{aligned}}$$

$$v(t) = \int a(t)\,dt$$

$$s(t) = \int v(t)\,dt$$

<u>Ex</u>: A ball is thrown straight up from the ground with initial velocity 3 m/s. Find height $h(t)$.

$$a(t) = -9.8 \quad \text{(gravity)}$$

$$v(t) = \int a(t)\,dt$$

$$v(t) = \int -9.8\,dt$$

$$\boxed{v(t) = -9.8t + C}$$

$v(0) = 3$: $\quad 3 = C$

$$\boxed{v(t) = -9.8t + 3}$$

$$h(t) = \int v(t)\,dt$$

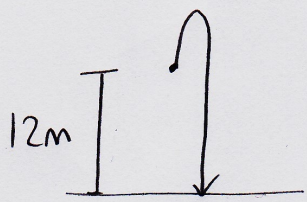
$$h(t) = \int (-9.8t + 3)\,dt$$

$$\boxed{h(t) = -4.9t^2 + 3t + C_1}$$ ← new name

$h(0)=0$: $\quad 0 = C_1$ ↗

$$h(t) = -4.9t^2 + 3t$$

<u>Ex</u>:



Ball is thrown straight up from 12m high. Takes 6s to land. Find $h(t)$ and initial velocity v_0

$$a(t) = -9.8 \quad \text{(gravity)}$$

$$v(t) = \int -9.8\,dt$$

$$\boxed{v(t) = -9.8t + C_1}$$

$v(0)=v_0$: $\quad v_0 = C_1$ ↗

$$\boxed{v(t) = -9.8t + v_0}$$

$$h(t) = \int (-9.8t + v_0)\,dt$$

$$\boxed{h(t) = -4.9t^2 + v_0 t + C_2}$$

$h(0)=12$: $\quad 12 = C_2$ ↗

$$\boxed{h(t) = -4.9t^2 + v_0 t + 12}$$

Given $h(6)=0$: $\quad 0 = -4.9(36) + 6v_0 + 12$

$$v_0 = 27.4 \text{ m/s}$$

<u>Ex</u>: Car travels in straight line with $a(t) = -4t \text{ m/s}^2$ (slowing down). Brakes are applied when velocity $= 20$ m/s. Stopping distance?

$$\boxed{\begin{array}{l} t=0 : \text{brakes applied} \\ v(0) = 20 \text{ m/s} \\ s(0) = 0 \end{array}}$$

1) Find $v(t)$, $s(t)$

$$a(t) = -4t$$

$$v(t) = \int -4t\,dt$$

$$= -2t^2 + C_1$$

$v(0)=20$: $\quad 20 = C_1$

$$\boxed{v(t) = -2t^2 + 20}$$

$$s(t) = \int (-2t^2 + 20)\,dt$$

$$= -\frac{2t^3}{3} + 20t + C_2 \leftarrow \text{new name}$$

$s(0) = 0$: $\quad 0 = C_2$

$$\boxed{s(t) = \frac{-2t^3}{3} + 20t}$$

2) Stopping time?

Set $v(t) = 0$

$$-2t^2 + 20 = 0$$

$$t^2 = 10$$

$$t = \pm\sqrt{10}$$

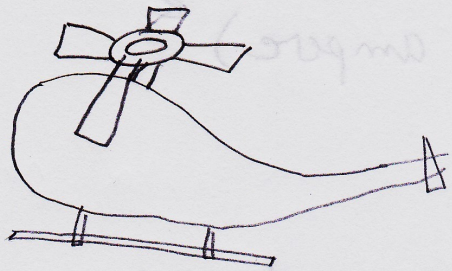
$$t = \sqrt{10}$$

3) Stopping distance

$$s(\sqrt{10}) = \frac{-2\sqrt{10}^3}{3} + 20\sqrt{10}$$

$$\approx 42.2\,m$$

Ex:



Angular velocity of rotor is $\frac{d\theta}{dt} = \sqrt{(4t+1)^3}$

Find angular displacement $\theta(t)$ after 5s.

$$\theta = \int \frac{d\theta}{dt}\, dt$$

$$= \int (4t+1)^{3/2} dt$$

$$\boxed{\begin{aligned} u &= 4t+1 \\ du &= 4dt \\ \frac{du}{4} &= dt \end{aligned}}$$

$$= \frac{1}{4}\int u^{3/2} du$$

$$= \frac{1}{4}\left(\frac{2}{5}u^{5/2}\right) + C$$

$$\boxed{\theta = \frac{1}{10}(4t+1)^{5/2} + C}$$

$\theta(0)=0$:

$$0 = \frac{1}{10} + C$$

$$C = \frac{-1}{10}$$

$$\theta = \frac{1}{10}(4t+1)^{5/2} - \frac{1}{10}$$

$$\theta(5) = \frac{1}{10}(21)^{5/2} - \frac{1}{10}$$

$$\approx 201.99 \text{ rad}$$

$(\div 2\pi)$ or ≈ 32 full rotations

q: charge (C for Coulomb) ← units

i: current (A for ampere) ↙

$$\boxed{q = \int i\,dt}$$

<u>Ex</u>: Given $i(t) = t\sqrt{t^2+4}$ and $q(0)=0$, find $q(3)$

→ total charge past a point after 3s

$$q = \int i\,dt$$

$$q = \int t\sqrt{t^2+4}\,dt$$

← $\boxed{\begin{aligned} u &= t^2+4 \\ du &= 2t\,dt \\ \frac{du}{2} &= t\,dt \end{aligned}}$

$$= \frac{1}{2}\int u^{1/2}\,du$$

$$= \frac{1}{2}\left(\frac{2}{3}u^{3/2}\right) + C$$

$$= \frac{1}{3}(t^2+4)^{3/2} + C$$

$q(0)=0$:

$$0 = \frac{1}{3}(4)^{3/2} + C$$

$$C = -8/3$$

$$q = \frac{1}{3}(t^2+4)^{3/2} - \frac{8}{3}$$

$$q(3) \approx 13\text{ C}$$

V: voltage (V for volt) ← units

C: capacitance (F for farad) ←

$$\boxed{V = \frac{1}{C}\int i\,dt}$$

<u>Ex</u>: Given $i = 3.0 \times 10^{-2}$ A

and $C = 4.0 \times 10^{-6}$ F (both constant),

find V after 1.0×10^{-3} s if $V(0) = 0$.

$$V = \frac{1}{C}\int i\,dt$$

$$= \frac{1}{4.0 \times 10^{-6}} \int 3.0 \times 10^{-2}\,dt$$

$$= \frac{1}{4.0 \times 10^{-6}} \left(3.0 \times 10^{-2}\, t\right) + C_1$$

$V(0) = 0$: $0 = C_1$

$$V = \frac{3.0 \times 10^{-2}\, t}{4.0 \times 10^{-6}}$$

$$V(1.0 \times 10^{-3}) = \frac{3.0 \times 10^{-2}\,(1.0 \times 10^{-3})}{4.0 \times 10^{-6}}$$

$$= 7.5\text{ V}$$

L: inductance units: H for henry

$$\boxed{\frac{V}{L} = \frac{di}{dt}}$$

<u>Ex</u>: Given $V = 6 - 0.1t$ for a 2H-inductor, find current after 15s if initial current was zero.

$L = 2H$

$$\frac{di}{dt} = \frac{V}{L}$$

$$\frac{di}{dt} = \frac{6 - 0.1t}{2} = 3 - 0.05t$$

$$i = \int \frac{di}{dt}\, dt$$

$$= \int (3 - 0.05t)\, dt$$

$$= 3t - 0.025t^2 + C_1$$

i(0)=0 : $0 = C_1$ ↗

$$i = 3t - 0.025t^2$$

$$i(15) = 45 - 0.025(15)^2$$

$$= 39 \text{ A}$$