24.5 Sketching Polynomials

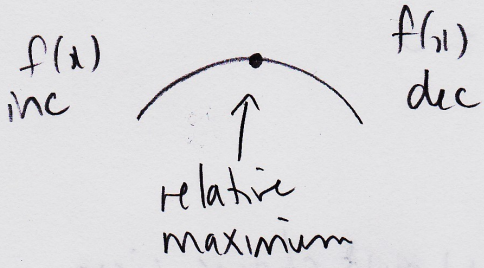
Will be useful for section 24.7

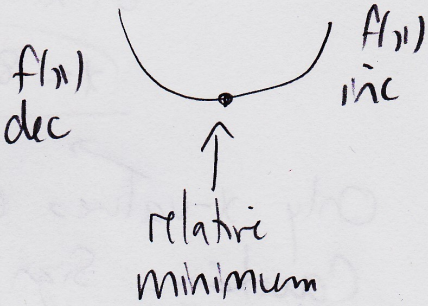
$f'(x) > 0$

$f(x)$ is increasing

$f'(x) < 0$

$f(x)$ is decreasing

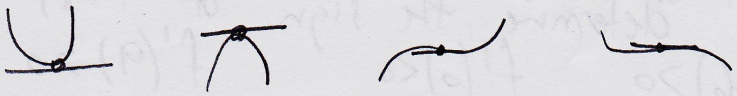




x-values where $f'(x) = 0$ are called <u>critical points</u>

They are <u>possible</u> relative max or relative min points

Critical points can look like:



Ex: Find all relative maximum and relative minimum points

$$f(x) = 2x^3 - 9x^2 - 240x + 5$$

→ Set $f'(x) = 0$

$$f'(x) = 6x^2 - 18x - 240$$

$$6x^2 - 18x - 240 = 0$$

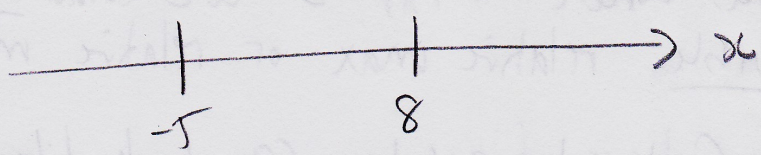
$$6(x^2 - 3x - 40) = 0$$

$$6(x-8)(x+5) = 0$$

$$x = 8, -5$$

Only x-values where $f'(x)$ might change sign

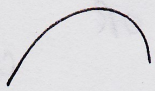
Caution: Sign change is not guaranteed



Test an x-value in each interval to determine the sign of $f'(x)$

$f'(-6) > 0$ $\quad f'(0) < 0$ $\quad f'(9) > 0$

		-5		8	
$f'(x)$	$+$		$-$		$+$
$f(x)$	INC		DEC		INC



Relative maximum at $x=-5$

$(-5, 730)$

$\uparrow$

$f(-5)$

Relative minimum at $x=8$

$(8, -1467)$

$\uparrow$

$f(8)$

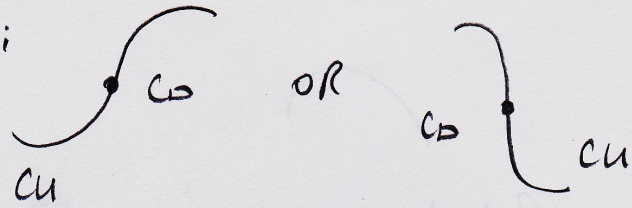
$f''(x) > 0$

$f(x)$ is "concave up"

$f''(x) < 0$

$f(x)$ is "concave down"

Points of inflection:



$$\boxed{\text{Points where } f''(x) = 0 \text{ are } \underline{\text{possible}} \text{ points of inflection}}$$

<u>Ex</u>: Find all points of inflection for

$$f(x) = 2x^3 - 9x^2 - 240x + 15$$

→ Set $f''(x) = 0$

$$f'(x) = 6x^2 - 18x - 240$$

$$f''(x) = 12x - 18$$

$$12x - 18 = 0$$

$$12x = 18$$

$$x = \frac{18}{12} \text{ or } \frac{3}{2}$$

$x = \frac{3}{2}$

Only x-value where $f''(x)$ might change sign

<u>Caution</u>: Sign change is not guaranteed

Test an x-value in each interval to determine the sign of $f''(x)$

		$\frac{3}{2}$	
$f''(x)$	$\ominus$		$\oplus$
$f(x)$	CD		CU

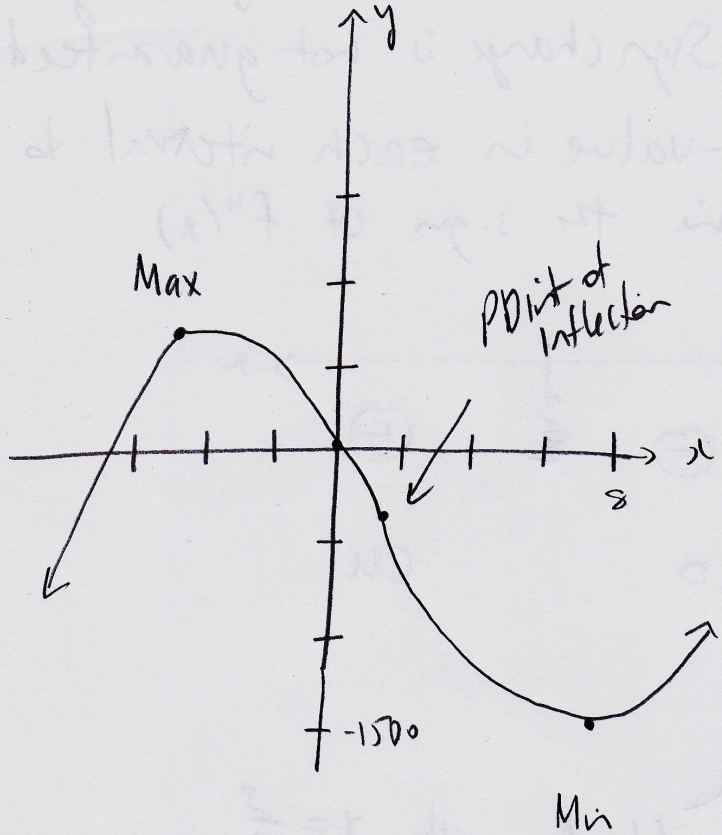
Point of inflection at $x = \frac{3}{2}$

$\left(\frac{3}{2}, -368.5\right)$

$f\left(\frac{3}{2}\right)$

$$\boxed{\begin{array}{l} \text{y-intercept}: \quad x = 0 \\ \qquad\qquad\quad f(0) = 5 \\ \qquad\qquad\quad (0, 5) \end{array}}$$

Sketch $f(x) = 2x^3 - 9x^2 - 240x + 5$:



Ex: $f(x) = x^6 - 15x^4$

a) Find y-intercept
b) all relative max/min
c) all points of inflection
d) sketch

a) Set $x=0$:

$f(0) = 0$

$(0,0)$

b) Set $f'(x)=0$ and make table

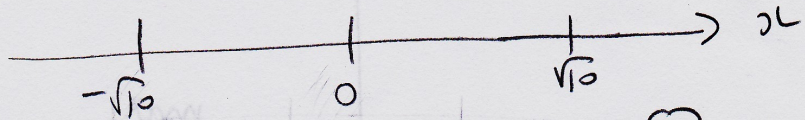
$f'(x) = 6x^5 - 60x^3$

$6x^5 - 60x^3 = 0$

$6x^3(x^2-10) = 0$

$x=0$ $\quad$ $x^2 = 10$

$x = \pm\sqrt{10}$



$f'(x)$	$\ominus$	$\oplus$	$\ominus$	$\oplus$
$f(x)$	DEC	INC	DEC	INC

$\leftarrow f(-\sqrt{10})$

relative min $(-\sqrt{10}, -500)$

$\leftarrow f(0)$

relative max $(0, 0)$

$\leftarrow f(\sqrt{10})$

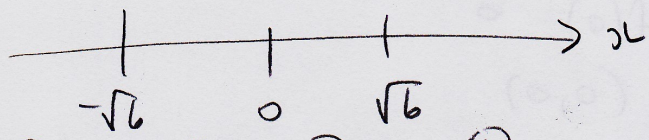
relative min $(\sqrt{10}, -500)$

c) Set $f''(x)=0$ and make table

$$f'(x) = 6x^5 - 60x^3$$
$$f''(x) = 30x^4 - 180x^2$$
$$30x^4 - 180x^2 = 0$$
$$30x^2(x^2-6) = 0$$

$x = 0$ $\quad$ $x^2 = 6$

$x = \pm\sqrt{6}$



$f''(x)$	$\oplus$	$\ominus$	$\ominus$	$\oplus$
$f(x)$	CU	CD	CD	CU

Points of inflection $(-\sqrt{6}, -324)$ ← $f(-\sqrt{6})$

$(\sqrt{6}, -324)$ ← $f(\sqrt{6})$

d) Sketch

$\sqrt{10} \approx 3.2$

$\sqrt{6} \approx 2.4$