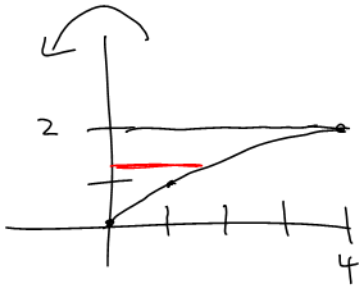
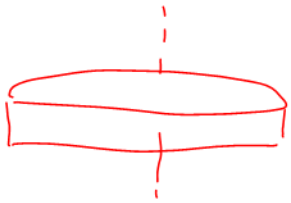
Name:_____

Consider the first-quadrant region bounded by $x = 0, y = \sqrt{x}$ and $y = 2$. Revolve the region about the y-axis. Find the volume of the resulting solid.

Disk Method





$$dV = \pi x^2 dy$$

$y = \sqrt{x}$

$\sqrt{x} = y$

$x = y^2$

$x^2 = y^4$

$$V = \pi \int_0^2 x^2 dy$$

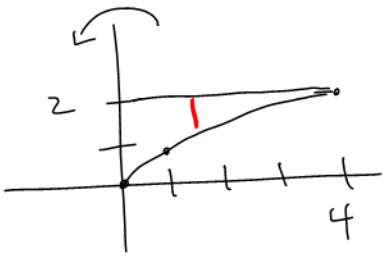
$$= \pi \int_0^2 y^4 dy$$

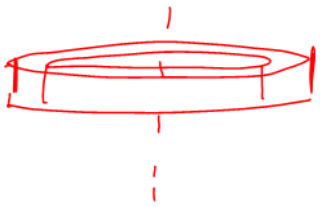
$$= \pi \left[\frac{y^5}{5} \right]_0^2$$

$$= \frac{32\pi}{5}$$

Shell Method on next page →

Shell Method





$$dV = 2\pi(\text{radius})(\text{height})(\text{thickness})$$

$$= 2\pi x\,(2-\sqrt{x})\,dx$$

$$V = 2\pi \int_0^4 x(2-\sqrt{x})\,dx$$

$$= 2\pi \int_0^4 (2x - x^{3/2})\,dx$$

$$= 2\pi \left[x^2 - \frac{2}{5}x^{5/2} \right]_0^4$$

$$= 2\pi \left[16 - \frac{64}{5} \right]$$

$$= 2\pi \left(\frac{16}{5} \right)$$

$$= \frac{32\pi}{5}$$