Math 191 Test Three

Time: 50 minutes

Total: 20 marks

Name: _____

1. [3 marks] Find $\frac{dy}{dx}$:

a) $y = \ln(x^9 + 1)$

$$\frac{dy}{dx} = \frac{1}{x^9+1}(9x^8)$$

$$= \frac{9x^8}{x^9+1}$$

b) $y = 2^{6x+1}$

$$\frac{dy}{dx} = (\ln 2)2^{6x+1}(6)$$

$$= (6\ln 2)2^{6x+1}$$

c) $y = \sin^{-1}(4x)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(4x)^2}}(4)$$

$$= \frac{4}{\sqrt{1-16x^2}}$$

2. [4 marks] Use Simpson's Rule with four intervals to approximate $\int_3^7 \sqrt{1+x^2}\,dx$. Give your answer to two decimal places.

$\frac{b-a}{n} = \frac{7-3}{4} = 1$ step size

x	$y = \sqrt{1+x^2}$
3	$\sqrt{10}$
4	$\sqrt{17}$
5	$\sqrt{26}$
6	$\sqrt{37}$
7	$\sqrt{50}$

$$\text{Integral} \approx \frac{b-a}{3n}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$\approx \frac{1}{3}\left[\sqrt{10} + 4\sqrt{17} + 2\sqrt{26} + 4\sqrt{37} + \sqrt{50}\right]$$

$$\approx 20.42$$

3. [3 marks] Evaluate $\int_1^2 (3x^3 + x^7)\,dx$

$$= \left[\frac{3x^4}{4} + \frac{x^8}{8}\right]_1^2$$

$$= 44 - \left(\frac{3}{4} + \frac{1}{8}\right)$$

$$= \frac{345}{8} \text{ or } 43.125$$

4. [3 marks] Find $\int (2x^3+1)\sqrt{x^4+2x}\,dx$

$u = x^4+2x$

$du = (4x^3+2)\,dx$

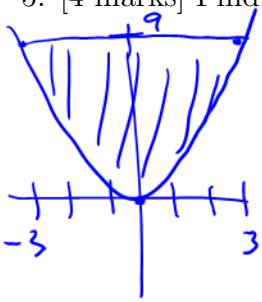
$\frac{du}{2} = (2x^3+1)\,dx$

$\text{Integral} = \int \frac{\sqrt{u}\,du}{2}$

$= \frac{1}{3}u^{3/2} + C$

$= \frac{1}{3}(x^4+2x)^{3/2} + C$

5. [4 marks] Find the area bounded by $y = x^2$ and $y = 9$



$y = y$

$x^2 = 9$

$x = \pm 3$

$A = \int (y_t - y_b)\,dx$

$= \int_{-3}^{3} (9 - x^2)\,dx$

$= \left[9x - \frac{x^3}{3}\right]_{-3}^{3}$

$= 18 - (-18)$

$= 36$

6. [3 marks] Find the value of $f(t)$ when $t = 2$ given:
$\frac{df}{dt} = 4t^3$ and
$f(t) = 6$ when $t = 1$

$$f(t) = \int 4t^3 \, dt$$

$$\boxed{f(t) = t^4 + C}$$

Sub $f = 6$, $t = 1$: $6 = 1 + C$

$C = 5$

$$\boxed{f(t) = t^4 + 5}$$

$f(2) = 21$