Math 191 Test Two

Time: 50 minutes

Total: 20 marks

Name: ______________________________

1. [2 marks] We want to approximate a solution to $3x^4 - 18x^2 + 5 = 0$. Use the formula below with $x_0 = 2$ to find x_1. Round your answer to two decimal places.

$f(x) = 3x^4 - 18x^2 + 5$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f'(x) = 12x^3 - 36x$

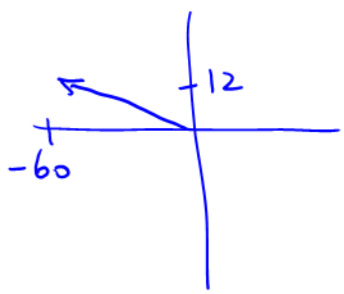
x_n	$f(x_n)$	$f'(x_n)$	$x_n - \frac{f(x_n)}{f'(x_n)}$
2	-19	24	2.79

$x_1 \simeq 2.79$

2. [4 marks] An object's position (in metres) after t seconds is described by: $x = 20t^{-3} + 6$, $y = 4t^2 + 4t$. Find the object's velocity at $t = 1$ second. Remember to include speed and direction. Round your values to one decimal place.

$v_x = -60t^{-4}$ $\quad v_y = 8t + 4$

@ $t=1$: $v_x = -60$ $\quad v_y = 12$



$v = \sqrt{(-60)^2 + 12^2}$

$\simeq 61.2$ m/s

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$ $(+180°?)$

$= \tan^{-1}\left(\frac{12}{-60}\right) + 180°$

$\simeq 168.7°$

3. [3 marks] Let $y = 8x^3 - x^4$. For which x-values is y increasing?

$$y' = 24x^2 - 4x^3$$

Set $y' = 0$:

$$24x^2 - 4x^3 = 0$$

$$4x^2(6 - x) = 0$$

$$x = 0, 6$$

		0		6	
y'	$+$		$+$		$-$
y	INC		INC		DEC

y is increasing for $x < 6$

4. [3 marks] Use the formula below to find the linearization of $f(x) = \sqrt{x}$ using the value $a = 16$.

$f(x) \approx f(a) + f'(a)(x - a)$

$$f(x) = x^{1/2} \qquad f(16) = 4$$

$$f'(x) = \frac{1}{2}x^{-1/2} \qquad f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{8}$$

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$\sqrt{x} \approx 4 + \frac{1}{8}(x-16)$$

Valid near $x = 16$

5. [4 marks] Find y':

a) $y = \sin^3 x$

$y = [\sin x]^3$

$y' = 3\sin^2 x \cos x$

b) $y = \sin x^3$

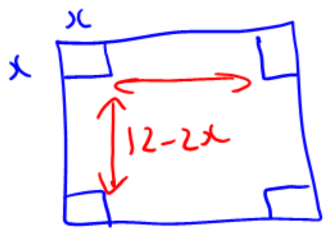
$y' = 3x^2 \cos x^3$

c) $y = \csc(2 + 7x)$

$y' = -\csc(2+7x)\cot(2+7x) \cdot 7$

$= -7\csc(2+7x)\cot(2+7x)$

6. [4 marks] We cut the corners from a 12cm x 12cm metal sheet to form an open-topped box. Find the height of the box that maximizes the box's volume.



$$V = x(12-2x)^2$$
$$V = x(144 - 48x + 4x^2)$$
$$V = 144x - 48x^2 + 4x^3$$

$$V' = 144 - 96x + 12x^2$$

Set $V' = 0$:

$$144 - 96x + 12x^2 = 0$$
$$12(x^2 - 8x + 12) = 0$$
$$12(x-2)(x-6) = 0$$

$x = 2$ MAX

$x = 6$ MIN (Box has zero volume)

Answer: $x = 2$cm

Alternatively, if you did product rule:

$$V = x(12-2x)^2$$
$$V' = x \cdot 2(12-2x)(-2) + (12-2x)^2$$

Set $V' = 0$:

$$(12-2x)(-4x + 12 - 2x) = 0$$
$$(12-2x)(12-6x) = 0$$

MIN $x = 6$ $\qquad$ $x = 2$ MAX