Math 191 Test One

Time: 50 minutes

Total: 23 marks

Name: ______________________________

1. [3 marks] An object's displacement (in metres) is given by $s(t) = 2.8t^3 - 1.7t^2 + 2.5$, where t is measured in seconds. Find the object's acceleration at $t = 1.2$ seconds.

$v(t) = 8.4t^2 - 3.4t$

$a(t) = 16.8t - 3.4$

$a(1.2) = 16.76 \ m/s^2$

2. [3 marks] Evaluate the following limit:
$\lim_{x \to -3} \frac{9x+27}{x^2-2x-15}$

$$= \lim_{x \to -3} \frac{9\cancel{(x+3)}}{\cancel{(x+3)}(x-5)}$$

$$= \lim_{x \to -3} \frac{9}{x-5}$$

$$= \frac{9}{-8}$$

$$= \frac{-9}{8}$$

3. [5 marks] Use the **limit definition** to find $f'(x)$ for $f(x) = 6x^2 - 7$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{6(x+h)^2 - 7 - (6x^2 - 7)}{h}$$

$$= \lim_{h \to 0} \frac{6(x^2 + 2xh + h^2) \cancel{-7} - 6x^2 + \cancel{7}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{6x^2} + 12xh + 6h^2 - \cancel{6x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(12x + 6h)}{\cancel{h}}$$

$$= 12x$$

4. [4 marks] Find $f'(x)$ and **simplify fully**:

$f(x) = \frac{2x^2-3}{5-7x^2}$

$$\boxed{\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}}$$

$$f'(x) = \frac{(5-7x^2)(4x) \cancel{-} + 14x(2x^2-3)\cancel{(-14x)}}{(5-7x^2)^2}$$

$$= \frac{20x - 28x^3 + 28x^3 - 42x}{(5-7x^2)^2}$$

$$= \frac{-22x}{(5-7x^2)^2}$$

5. [4 marks] Find $\frac{dy}{dx}$ given:

$$8x^3 - \underbrace{9x^2y}_{\text{product rule}} + 4y^2 = 17$$

$$24x^2 - 9x^2\frac{dy}{dx} - 18xy + 8y\frac{dy}{dx} = 0$$

$$\left[-9x^2 + 8y\right]\frac{dy}{dx} = -24x^2 + 18xy$$

$$\frac{dy}{dx} = \frac{-24x^2 + 18xy}{-9x^2 + 8y}$$

OR $$\frac{dy}{dx} = \frac{24x^2 - 18xy}{9x^2 - 8y}$$

6. [4 marks] Find $\frac{dy}{dx}$:

a) $y = \frac{-2}{(3x+8)^5}$

$$y = -2(3x+8)^{-5}$$

$$\frac{dy}{dx} = 10(3x+8)^{-6}(3)$$

$$= \frac{30}{(3x+8)^6}$$

b) $y = 9\sqrt{4x^2+7}$

$$y = 9(4x^2+7)^{1/2}$$

$$\frac{dy}{dx} = \frac{9}{2}(4x^2+7)^{-1/2}(8x)$$

$$= \frac{36x}{\sqrt{4x^2+7}}$$