41.

$$\bar{x} = \frac{1}{A} \int_A x_e \, dA$$

$$dA = (top - bottom)\, dx$$

$$dA = (x - x^2)\, dx$$

$$A = \int_A dA$$

$$= \int_0^1 (x - x^2)\, dx$$

$$= \left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1$$

$$A = \frac{1}{6}$$
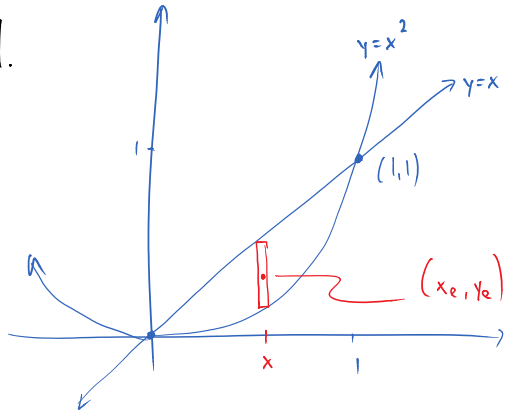
$$x_e = x$$

$$y_e = midpoint$$
$$= \frac{top + bottom}{2}$$

$$y_e = \frac{x + x^2}{2}$$

$$\bar{x} = \frac{1}{A} \int_A x_e \, dA$$

$$= \frac{6}{1} \int_0^1 x(x - x^2)\, dx$$

$$= 6 \int_0^1 (x^2 - x^3)\, dx$$

$$= 6\left(\frac{1}{3}x^3 - \frac{1}{4}x^4\right)\Big|_0^1$$

$$= 6\left(\frac{1}{12}\right)$$

$$= \frac{1}{2}$$

$$\bar{y} = \frac{1}{A} \int_A y_e \, dA$$

$$= \frac{6}{1} \int_0^1 \left(\frac{x + x^2}{2}\right)(x - x^2)\, dx$$
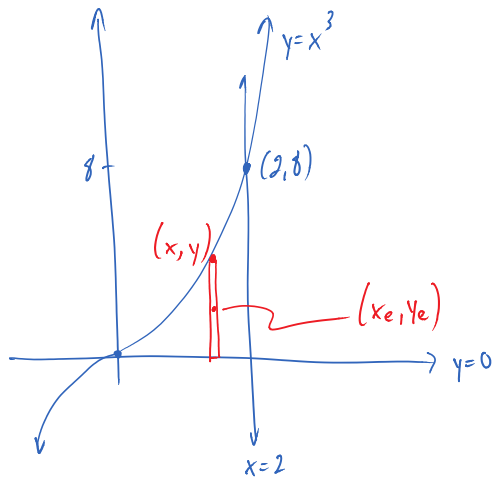
$$= 3 \int_0^1 (x^2 - x^4)\, dx$$

$$= 3\left(\frac{1}{3}x^3 - \frac{1}{5}x^5\right)\Big|_0^1$$

$$= 3\left(\frac{2}{15}\right)$$

$$= \frac{2}{5}$$

$$(\bar{x}, \bar{y}) = \left(\frac{1}{2}, \frac{2}{5}\right)$$

42.

$$\bar{x} = \frac{1}{A} \int_A x_e \, dA$$

$$dA = y \, dx$$
$$dA = x^3 \, dx$$

$$A = \int_A dA$$

$$= \int_0^2 x^3 \, dx$$

$$= \frac{1}{4} x^4 \Big|_0^2$$

$$\boxed{A = 4}$$

$x_e = x$

$y_e = \frac{1}{2} y = \frac{1}{2} x^3$

$$\bar{x} = \frac{1}{A} \int_A x_e \, dA$$

$$= \frac{1}{4} \int_0^2 x \cdot x^3 \, dx$$

$$= \frac{1}{4} \int_0^2 x^4 \, dx$$

$$= \frac{1}{4} \cdot \frac{1}{5} x^5 \Big|_0^2$$

$$= \frac{8}{5}$$

$$\bar{y} = \frac{1}{A} \int_A y_e \, dA$$

$$= \frac{1}{4} \int_0^2 \frac{1}{2} x^3 \cdot x^3 \, dx$$

$$= \frac{1}{8} \int_0^2 x^6 \, dx$$

$$= \frac{1}{8} \cdot \frac{1}{7} x^7 \Big|_0^2$$

$$= \frac{16}{7}$$

$$\left(\bar{x}, \bar{y} \right) = \left(\frac{8}{5}, \frac{16}{7} \right)$$

43.

$$SA = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$y = x^3$$
$$\frac{dy}{dx} = 3x^2$$
$$\left(\frac{dy}{dx}\right)^2 = (3x^2)^2 = 9x^4$$

$$SA = 2\pi \int_1^2 x^3 \sqrt{1 + 9x^4} \, dx$$

let $\quad u = 1 + 9x^4$
$$du = 36x^3 \, dx$$
$$\frac{1}{36} du = x^3 \, dx$$

$$= \frac{2\pi}{36} \int_{x=1}^{x=2} u^{1/2} \, du$$

$$= \frac{\pi}{18} \cdot \frac{2}{3} u^{3/2} \Big|_{x=1}^{x=2}$$

$$= \frac{\pi}{27} \left(1 + 9x^4\right)^{3/2} \Big|_1^2 \quad \approx 199.48$$

44.

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$y = 1 + 2x^{3/2}$$
$$\frac{dy}{dx} = 3x^{1/2}$$
$$\left(\frac{dy}{dx}\right)^2 = (3x^{1/2})^2 = 9x$$

$$s = \int_0^1 \sqrt{1 + 9x} \, dx$$

$$= \int_0^1 \underline{\left(1 + 9x\right)}^{1/2} \, dx$$
$$\textcolor{red}{\text{linear}}$$

$$= \frac{1}{9} \cdot \frac{2}{3} \left(1 + 9x\right)^{3/2} \Big|_0^1 \quad \approx 2.27$$

45.

$$Y_{av} = \frac{1}{b-a} \int_a^b f(x)\, dx$$

$$T_{av} = \frac{1}{5-0} \int_0^5 \left(10 + 50e^{-2t}\right) dt \quad \text{linear}$$

$$= \frac{1}{5} \left(10t + 50\left(-\frac{1}{2}\right)e^{-2t}\right)\Big|_0^5 = 15.0\ °C$$

46. a)
$$AB = \begin{bmatrix} 1 & -4 \\ 2 & -6 \end{bmatrix}\begin{bmatrix} -2 & 2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} -14 & -18 \\ -22 & -26 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 1 & -4 \\ 2 & -6 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 13 & -42 \end{bmatrix}$$

b)
$$AB = \begin{bmatrix} 1 & -1 & 2 \\ 4 & -3 & 3 \end{bmatrix}\begin{bmatrix} -2 & 5 \\ 1 & 6 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 8 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 & 5 \\ 1 & 6 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 4 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 18 & -13 & 11 \\ 25 & -19 & 20 \\ 13 & -11 & 16 \end{bmatrix}$$

c)
$$AB = \begin{bmatrix} 1 & -1 & 2 \\ 4 & -3 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ 13 \\ -8 \end{bmatrix} = \begin{bmatrix} -20 \\ -27 \end{bmatrix}$$

$$BA = \begin{bmatrix} 9 \\ 13 \\ -8 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 4 & -3 & 3 \end{bmatrix} \quad \text{undefined}$$

3×1 2×3

47. a) $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \dfrac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ if $ad-bc \neq 0$

$A = \begin{bmatrix} 1 & -6 \\ 4 & -7 \end{bmatrix}$

$\det(A) = \dfrac{1}{1(-7)-4(-6)} \begin{bmatrix} -7 & 6 \\ -4 & 1 \end{bmatrix}$

$= \dfrac{1}{17} \begin{bmatrix} -7 & 6 \\ -4 & 1 \end{bmatrix}$

b) $B = \begin{bmatrix} 12 & -3 \\ -8 & 2 \end{bmatrix}$

$ad - bc = 12(2) - (-3)(-8) = 0$

B^{-1} does not exist

48.

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 & 1 & 0 \\ 2 & 2 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - R_1} \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\downarrow R_1 - R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & -1 & 0 \\ 0 & 5 & 1 & -1 & 2 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right] \xleftarrow{R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & -1 & 0 \\ 1 & 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\downarrow R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 5 & 1 & -1 & 2 & 0 \end{array}\right] \xrightarrow{R_3 - 5R_2} \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & -4 & 4 & 2 & -5 \end{array}\right]$$

$$\downarrow R_1 + 2R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -1 & -1 & 2 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & -\frac{1}{2} & \frac{5}{4} \end{array}\right] \xleftarrow{-\frac{1}{4}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -1 & -1 & 2 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & -4 & 4 & 2 & -5 \end{array}\right]$$

$$\begin{array}{c} \downarrow R_1 - 2R_3 \\ \downarrow R_2 - R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{4} \\ 0 & 0 & 1 & -1 & -\frac{1}{2} & \frac{5}{4} \end{array}\right] \qquad \text{so} \qquad A^{-1} = \left[\begin{array}{ccc} 1 & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{4} \\ -1 & -\frac{1}{2} & \frac{5}{4} \end{array}\right]$$

49. a)

$$\begin{bmatrix} 8 & -1 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 17 \\ -5 \end{bmatrix}$$

$$\quad A \qquad\quad X \qquad\quad C$$

$$A^{-1} = \frac{1}{8-4} \begin{bmatrix} 1 & 1 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ 1 & 2 \end{bmatrix}$$

$$X = A^{-1}C = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 17 \\ -5 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$x = 3, \ y = 7$$

b)

$$\begin{bmatrix} 4 & 0 & 4 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \\ -2 \end{bmatrix}$$

$$\quad A \qquad\qquad X \qquad\quad C$$

$$\left[\begin{array}{ccc|ccc} 4 & 0 & 4 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 4 & 0 & 4 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\Big| \ R_2 - 4R_1$$
$$\Big\downarrow \ R_3 - R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & -\frac{1}{4} & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right] \xleftarrow[\ -R_3\ ]{-\frac{1}{4}R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & -4 & -4 & 1 & -4 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array}\right]$$

$$\Big| \ R_1 - 2R_3$$
$$\Big\downarrow \ R_2 - R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & -\frac{1}{4} & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{4} & -1 & 1 \\ 0 & 1 & 0 & -\frac{1}{4} & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array}\right]$$

$$X = A^{-1}C = \begin{bmatrix} \frac{1}{4} & -1 & 1 \\ -\frac{1}{4} & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ -4 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

$$x = 1, \ y = -1, \ z = -2$$

50. a)

$$\begin{bmatrix} 1 & 3 & -2 & | & 9 \\ 2 & -1 & 4 & | & 6 \\ -3 & 2 & -3 & | & -1 \end{bmatrix} \xrightarrow[R_3 + 3R_1]{R_2 - 2R_1} \begin{bmatrix} 1 & 3 & -2 & | & 9 \\ 0 & -7 & 8 & | & -12 \\ 0 & 11 & -9 & | & 26 \end{bmatrix}$$

$$\downarrow -\tfrac{1}{7} R_2$$

$$\begin{bmatrix} 1 & 0 & 10/7 & | & 27/7 \\ 0 & 1 & -8/7 & | & 12/7 \\ 0 & 0 & 25/7 & | & 50/7 \end{bmatrix} \xleftarrow[R_3 - 11R_2]{R_1 - 3R_2} \begin{bmatrix} 1 & 3 & -2 & | & 9 \\ 0 & 1 & -8/7 & | & 12/7 \\ 0 & 11 & -9 & | & 26 \end{bmatrix}$$

$$\downarrow \tfrac{7}{25} R_3$$

$$\begin{bmatrix} 1 & 0 & 10/7 & | & 27/7 \\ 0 & 1 & -8/7 & | & 12/7 \\ 0 & 0 & 1 & | & 2 \end{bmatrix} \xrightarrow[R_2 + \tfrac{8}{7} R_3]{R_1 - \tfrac{10}{7} R_3} \begin{bmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & 4 \\ 0 & 0 & 1 & | & 2 \end{bmatrix}$$

$$x = 1, \ y = 4, \ z = 2$$

b)

$$\begin{bmatrix} 3 & -18 & 21 & | & 12 \\ 2 & 7 & -6 & | & 3 \end{bmatrix} \xrightarrow{R_1 - R_2} \begin{bmatrix} 1 & -25 & 27 & | & 9 \\ 2 & 7 & -6 & | & 3 \end{bmatrix}$$

$$\downarrow R_2 - 2R_1$$

$$\begin{bmatrix} 1 & -25 & 27 & | & 9 \\ 0 & 1 & -\tfrac{20}{19} & | & -\tfrac{5}{19} \end{bmatrix} \xleftarrow{\tfrac{1}{57} R_2} \begin{bmatrix} 1 & -25 & 27 & | & 9 \\ 0 & 57 & -60 & | & -15 \end{bmatrix}$$

$$\downarrow R_1 + 25R_2$$

$$\begin{bmatrix} 1 & 0 & 13/19 & | & 46/19 \\ 0 & 1 & -20/19 & | & -5/19 \end{bmatrix}$$
$$\ \ \ x \quad\ \ y \quad\ \ z$$

$$x \ + \ \frac{13}{19} \, z \ = \ \frac{46}{19}$$

$$y \ - \ \frac{20}{19} \, z \ = \ -\frac{5}{19}$$

$$z = \text{anything}$$

let   z = t   then   the general solution is

$$\begin{cases} x = \dfrac{46}{19} - \dfrac{13}{19}t \\[2mm] y = -\dfrac{5}{19} + \dfrac{20}{19}t \\[2mm] z = t \end{cases}$$

if t = 0: $x = \dfrac{46}{19}$, $y = -\dfrac{5}{19}$, $z = 0$

if t = 1: $x = \dfrac{33}{19}$, $y = \dfrac{15}{19}$, $z = 1$

c)
$$\begin{bmatrix} 1 & 3 & 3 & | & 12 \\ 2 & 20 & 10 & | & 8 \\ 1 & 10 & 5 & | & 0 \end{bmatrix} \xrightarrow{R_2 - 2R_3} \begin{bmatrix} 1 & 3 & 5 & | & 12 \\ 0 & 0 & 0 & | & 8 \\ \end{bmatrix} \times$$

no solution