1. [4 marks] Simpson's Rule says:

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 4y_{n-1} + y_n].$$

Use Simpson's Rule with 4 intervals to approximate $\int_1^3 4^x\,dx$.

Round your final answer to two decimal places.

$n = 4$ is even ✓

$$\Delta x = \frac{b-a}{n} = \frac{3-1}{4} = 0.5$$

x	$y = 4^x$
1	4
1.5	8
2	16
2.5	32
3	64

$$\int_1^3 4^x\,dx \approx \frac{0.5}{3}[4 + 4(8) + 2(16) + 4(32) + 64]$$

$$\approx 43.33$$

2. [4 marks] Evaluate $\int_1^{16} (\frac{2}{x^2} + \frac{3}{\sqrt{x}})\, dx$

$$= \int_1^{16} (2x^{-2} + 3x^{-1/2})\, dx$$

$$= \left[-2x^{-1} + 6x^{1/2}\right]_1^{16}$$

$$= \left[-\frac{2}{16} + 6\sqrt{16}\right] - \left[-2 + 6\right]$$

$$= 19.875 \text{ or } \frac{159}{8}$$

3. [6 marks] Find y':

a) $y = \log_2(3x+1)$

$$y' = \frac{1}{\ln 2} \frac{1}{3x+1} (3)$$

$$= \frac{3}{(3x+1)\ln 2}$$

b) $y = 2^{3x+1}$

$$y' = \ln 2 \left(2^{3x+1}\right)(3)$$

$$= (3\ln 2)\, 2^{3x+1}$$

c) $y = 6\tan^{-1}(2\sqrt{x})$

$$y' = 6 \frac{1}{1+(2\sqrt{x})^2} \frac{d}{dx}(2x^{1/2})$$

$$= \frac{6}{1+4x} (x^{-1/2})$$

$$= \frac{6}{\sqrt{x}(1+4x)}$$

4. [3 marks] Find $\int \frac{7x}{(6x^2+1)^3}\,dx$

$$\boxed{\begin{aligned} u &= 6x^2+1 \\ \frac{du}{dx} &= 12x \\ du &= 12x\,dx \\ \frac{du}{12} &= x\,dx \end{aligned}}$$

$$= \frac{7}{12}\int \frac{du}{u^3}$$

$$= \frac{7}{12}\int u^{-3}\,du$$

$$= \frac{7}{12}\left(-\frac{1}{2}u^{-2}\right)+C$$

$$= -\frac{7}{24}(6x^2+1)^{-2}+C$$

5. [4 marks] An object is travelling in a straight line with $a(t) = 4\text{ m/s}^2$ and an initial velocity of 8 m/s. At what time is the object's displacement equal to 42 m?

$$\boxed{a(t) = 4}$$

$$v(t) = \int 4\,dt$$

$$v(t) = 4t + C_1$$

$t = 0$
$v(t) = 8$: $8 = C_1$

$$\boxed{v(t) = 4t + 8}$$

$$s(t) = \int (4t+8)\,dt$$

$$s(t) = 2t^2 + 8t + C_2$$

$s(0) = 0$ by definition of displacement

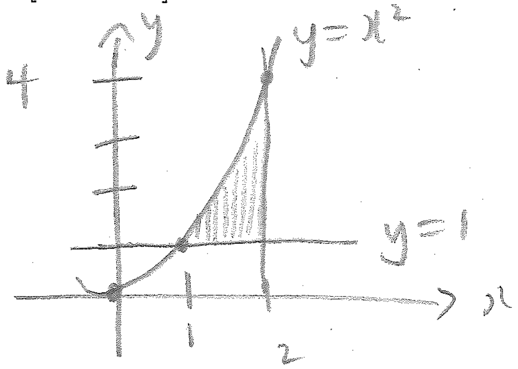
$s(t) = 0$
$t = 0$: $0 = C_2$

$$\boxed{s(t) = 2t^2 + 8t}$$

Set
$$42 = 2t^2 + 8t$$
$$0 = 2t^2 + 8t - 42$$
$$0 = 2(t^2 + 4t - 21)$$
$$0 = 2(t+7)(t-3)$$
$$t = -7, 3$$

$$\boxed{t = 3 \text{ seconds}}$$

6. [4 marks] Find the area bounded by $y = x^2, x = 2$ and $y = 1$.



$$A = \int_a^b (y_t - y_b)\,dx$$

$$= \int_1^2 (x^2 - 1)\,dx$$

$$= \left[\frac{x^3}{3} - x\right]_1^2$$

$$= \left(\frac{8}{3} - 2\right] - \left[\frac{1}{3} - 1\right]$$

$$= \frac{4}{3} \text{ or } 1.33$$

If you used horizontal slices:

$$A = \int_c^d (x_r - x_\ell)\,dy$$

$$= \int_1^4 (2 - \sqrt{y})\,dy$$

$$= \left[2y - \frac{2}{3}y^{3/2}\right]_1^4$$

$$= \left[8 - \frac{16}{3}\right] - \left[2 - \frac{2}{3}\right]$$

$$= \frac{4}{3}$$