Section 25.2

(17) $\int \left(\frac{t^2}{2} - \frac{2}{t^2} \right) dt$

$= \int \left(\frac{t^2}{2} - 2t^{-2} \right) dt$

$= \frac{t^3}{6} + 2t^{-1} + C$

or $\frac{t^3}{6} + \frac{2}{t} + C$

(19) $\int \sqrt{x}\,(x^2 - x)\,dx$

$= \int \left(x^{5/2} - x^{3/2} \right) dx$

$= \frac{2}{7} x^{7/2} - \frac{2}{5} x^{5/2} + C$

(21) $\int \left(2x^{-2/3} + 3^{-2}\right) dx$

$= \int \left(2x^{-2/3} + \frac{1}{9}\right) dx$

$= 6x^{1/3} + \frac{x}{9} + C$

(23) $\int (1 + 12x^2)^2 \, dx$

$= \int \left(1 + 24x^2 + 144x^4\right) dx$

$= x + 8x^3 + \frac{144x^5}{5} + C$

(25) $\int (x^2-1)^5 (2x\,dx)$

$$\boxed{\begin{aligned} u &= x^2-1 \\ du &= 2x\,dx \end{aligned}}$$

$= \int u^5\,du$

$= \frac{u^6}{6} + C$

$= \frac{(x^2-1)^6}{6} + C$

(27) $\int (x^4+3)^4 (4x^3\,dx)$

$$\boxed{\begin{aligned} u &= x^4+3 \\ du &= 4x^3\,dx \end{aligned}}$$

$= \int u^4\,du$

$= \frac{u^5}{5} + C$

$= \frac{(x^4+3)^5}{5} + C$

(29) $\int (2\theta^5 + 5)^7 \theta^4 d\theta$

$$\boxed{\begin{aligned} u &= 2\theta^5 + 5 \\ du &= 10\theta^4 d\theta \\ \frac{du}{10} &= \theta^4 d\theta \end{aligned}}$$

$$= \frac{1}{10}\int u^7 du$$

$$= \frac{1}{10}\left(\frac{u^8}{8}\right) + C$$

$$= \frac{(2\theta^5+5)^8}{80} + C$$

(31) $\int \sqrt{8x+1}\, dx$

$$\boxed{\begin{aligned} u &= 8x+1 \\ du &= 8\,dx \\ \frac{du}{8} &= dx \end{aligned}}$$

$$= \frac{1}{8}\int \sqrt{u}\, du$$

$$= \frac{1}{8}\left(\frac{2}{3}u^{3/2}\right) + C$$

$$= \frac{1}{12}\left(8x+1\right)^{3/2} + C$$

(33) $\int \frac{x\,dx}{\sqrt{6x^2+1}}$

$$u = 6x^2+1$$
$$du = 12x\,dx$$
$$\frac{du}{12} = x\,dx$$

$$= \frac{1}{12}\int \frac{du}{\sqrt{u}}$$

$$= \frac{1}{12}\int u^{-1/2}\,du$$

$$= \frac{1}{12}\left(2u^{1/2}\right) + C$$

$$= \frac{1}{6}\sqrt{6x^2+1} + C$$

(35)
$$\int \frac{4z-4}{\sqrt{z^2-2z}}\,dz$$

$$= \int \frac{4(z-1)}{\sqrt{z^2-2z}}\,dz$$

$$u = z^2 - 2z$$
$$du = (2z-2)\,dz$$
$$\frac{du}{2} = (z-1)\,dz$$

$$= \frac{4}{2}\int \frac{du}{\sqrt{u}}$$

$$= 2\int u^{-1/2}\,du$$

$$= 2\left(2u^{1/2}\right) + C$$

$$= 4\sqrt{z^2-2z} + C$$

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$\frac{dy}{dx} = 6x^2$ and curve passes through $(0, 2)$.
Find y.

$$\frac{dy}{dx} = 6x^2$$

$$dy = 6x^2\,dx$$

$$\int dy = \int 6x^2\,dx$$

$$\boxed{y = 2x^3 + C}$$

Sub $y = 2$, $x = 0$: $2 = C$

$$\boxed{y = 2x^3 + 2}$$

(39) $\frac{dy}{dx} = x^2(1-x^3)^5$ and curve passes through $(1,5)$.
Find y.

$$\frac{dy}{dx} = x^2(1-x^3)^5$$

$$dy = x^2(1-x^3)^5 dx$$

$$\int dy = \int x^2(1-x^3)^5 dx$$

$$y = \int x^2(1-x^3)^5 dx$$

$$\boxed{\begin{aligned} u &= 1-x^3 \\ du &= -3x^2 dx \\ -\tfrac{1}{3} du &= x^2 dx \end{aligned}}$$

$$y = \frac{-1}{3}\int u^5 du$$

$$y = \frac{-1}{3}\left(\frac{1}{6}u^6\right) + c$$

$$\boxed{y = \frac{-1}{18}(1-x^3)^6 + c}$$

Sub $y=5$, $x=1$: $5 = \frac{-1}{18}(0) + c$

$$c = 5$$

$$\boxed{y = \frac{-1}{18}(1-x^3)^6 + 5}$$

(47) $\frac{dy}{dx} = -x\sqrt{1-4x^2}$ and curve passes through $(0,7)$. Find y.

$$\frac{dy}{dx} = -x\sqrt{1-4x^2}$$

$$dy = -x\sqrt{1-4x^2}\,dx$$

$$\int dy = \int -x\sqrt{1-4x^2}\,dx$$

$$y = \int -x\sqrt{1-4x^2}\,dx$$

$$\boxed{\begin{aligned} u &= 1-4x^2 \\ du &= -8x\,dx \\ -\tfrac{1}{8}\,du &= x\,dx \end{aligned}}$$

$$y = \frac{1}{8}\int \sqrt{u}\,du$$

$$y = \frac{1}{8}\left(\frac{2}{3}u^{3/2}\right) + C$$

$$\boxed{y = \frac{1}{12}(1-4x^2)^{3/2} + C}$$

Sub $y=7$, $x=0$: $7 = \frac{1}{12}(1) + C$

$$C = 7 - \frac{1}{12}$$

$$= \frac{84}{12} - \frac{1}{12}$$

$$= \frac{83}{12}$$

$$\boxed{y = \frac{1}{12}(1-4x^2)^{3/2} + \frac{83}{12}}$$

OR $\boxed{12y = (1-4x^2)^{3/2} + 83}$

(55) $\frac{dT}{dr} = -4500(r+1)^{-3}$

Find T if $T = 2500°C$ for $r = 0$.

$$\frac{dT}{dr} = -4500(r+1)^{-3}$$

$$dT = -4500(r+1)^{-3}\,dr$$

$$\int dT = \int -4500(r+1)^{-3}\,dr$$

$$T = \int -4500(r+1)^{-3}\,dr$$

$$\boxed{\begin{aligned} u &= r+1 \\ du &= dr \end{aligned}}$$

$$T = \int -4500\,u^{-3}\,du$$

$$T = -4500\left(-\tfrac{1}{2}u^{-2}\right) + C$$

$$\boxed{T = 2250(r+1)^{-2} + C}$$

$T = 2500$, $r = 0$; $2500 = 2250(1) + C$

$$C = 250$$

$$T = 2250(r+1)^{-2} + 250$$