(3) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h) - 1 \ - 3x + 1}{h}$$

$$= \lim_{h \to 0} \frac{3x + 3h - 3x}{h}$$

$$= \lim_{h \to 0} \frac{3h}{h}$$

$$= \lim_{h \to 0} 3$$

$$= 3$$

(7) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 1 \ - x^2 + 1}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h)}{h}$$

$$= \lim_{h \to 0} 2x + h$$

$$= 2x$$

(11)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 7(x+h) - x^2 + 7x}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 7x - 7h - x^2 + 7x}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 - 7h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h - 7)}{h}$$

$$= \lim_{h \to 0} 2x + h - 7$$

$$= 2x - 7$$

(15) Let's calculate $(x+h)^3$ in advance.

$$\begin{aligned}(x+h)^3 &= (x+h)(x+h)^2 \\ &= (x+h)(x^2 + 2xh + h^2) \\ &= x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3 \\ &= x^3 + 3x^2h + 3xh^2 + h^3\end{aligned}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 + 4(x+h) - 3\pi - x^3 - 4x + 3\pi}{h}$$

$$= \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3+4x+4h-x^3-4x}{h}$$

$$= \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3+4h}{h}$$

$$= \lim_{h\to 0} \frac{h(3x^2+3xh+h^2+4)}{h}$$

$$= \lim_{h\to 0} 3x^2+3xh+h^2+4$$

$$= 3x^2+4$$

(19) $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[x+h+\frac{4}{3(x+h)}-x-\frac{4}{3x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[h+\frac{4x-4(x+h)}{3(x+h)x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[h+\frac{4x-4x-4h}{3(x+h)x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[h-\frac{4h}{3(x+h)x}\right]$$

$$= \lim_{h\to 0} 1-\frac{4}{3(x+h)x}$$

$$= 1-\frac{4}{3x^2}$$

(3)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[1 + \frac{2}{x+h} - 1 - \frac{2}{x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{2}{x+h} - \frac{2}{x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{2x - 2(x+h)}{(x+h)x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{2x - 2x - 2h}{(x+h)x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{-2h}{(x+h)x}\right]$$

$$= \lim_{h \to 0} \frac{-2}{(x+h)x}$$

$$= -\frac{2}{x^2}$$

$f'(x)$ is defined when $x \neq 0$

$f(x)$ is differentiable when $x \neq 0$

(35) $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 - 4(x+h) - x^2 + 4x}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 4x - 4h - x^2 + 4x}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 - 4h}{h}$$

$$= \lim_{h\to 0} \frac{h(2x+h-4)}{h}$$

$$= \lim_{h\to 0} 2x + h - 4$$

$$= 2x - 4$$

We want $2x - 4 = 6$

$2x = 10$

$x = 5$

$x = 5 \rightarrow y = x^2 - 4x$

$y = 5$

The point is $(x, y) = (5, 5)$.

(39) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{\left(\sqrt{x+h+1} - \sqrt{x+1}\right)}{h} \frac{\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}{\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}$$

$$= \lim_{h \to 0} \frac{x+h+1 - (x+1)}{h\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}$$

$$= \lim_{h \to 0} \frac{h}{h\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \frac{1}{\sqrt{x+1} + \sqrt{x+1}}$$

$$= \frac{1}{2\sqrt{x+1}}$$

$f'(x)$ is defined for $x > -1$

$f(x)$ is differentiable for $x > -1$