

## Section 16.3

$$\textcircled{3} \quad A = \begin{bmatrix} 2 & -5 \\ -2 & 4 \end{bmatrix}$$

$$\begin{aligned} \det A &= 2(4) - (-5)(-2) \\ &= -2 \end{aligned}$$

$$A^{-1} = \frac{-1}{2} \begin{bmatrix} 4 & 5 \\ 2 & 2 \end{bmatrix}$$

$$\textcircled{5} \quad A = \begin{bmatrix} -1 & 5 \\ 4 & 10 \end{bmatrix}$$

$$\begin{aligned} \det A &= -1(10) - (5)(4) \\ &= -30 \end{aligned}$$

$$A^{-1} = \frac{-1}{30} \begin{bmatrix} 10 & -5 \\ -4 & -1 \end{bmatrix}$$



$$(13) [A | I]$$

$$\left[ \begin{array}{cc|cc} 2 & 4 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{array} \right]$$

$$\frac{R_1}{2} \left[ \begin{array}{cc|cc} 1 & 2 & \frac{1}{2} & 0 \\ -1 & -1 & 0 & 1 \end{array} \right]$$

$$R_2 + R_1 \left[ \begin{array}{cc|cc} 1 & 2 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} & 1 \end{array} \right]$$

$$R_1 - 2R_2 \left[ \begin{array}{cc|cc} 1 & 0 & -\frac{1}{2} & -2 \\ 0 & 1 & \frac{1}{2} & 1 \end{array} \right]$$

$$\begin{array}{cc} \uparrow & \uparrow \\ I & A^{-1} \end{array}$$

(19)

$[A | I]$

$$\left[ \begin{array}{ccc|ccc} 1 & -3 & -2 & 1 & 0 & 0 \\ -2 & 7 & 3 & 0 & 1 & 0 \\ 1 & -1 & -3 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} R_2 + 2R_1 \\ R_3 - R_1 \end{array} \left[ \begin{array}{ccc|ccc} 1 & -3 & -2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 2 & 1 & 0 \\ 0 & 2 & -1 & -1 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} R_1 + 3R_2 \\ R_3 - 2R_2 \end{array} \left[ \begin{array}{ccc|ccc} 1 & 0 & -5 & 7 & 3 & 0 \\ 0 & 1 & -1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -5 & -2 & 1 \end{array} \right]$$

$$\begin{array}{l} R_1 + 5R_3 \\ R_2 + R_3 \end{array} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & -18 & -7 & 5 \\ 0 & 1 & 0 & -3 & -1 & 1 \\ 0 & 0 & 1 & -5 & -2 & 1 \end{array} \right]$$

↑  
I

↑  
 $A^{-1}$

(21)

$$\left[ \begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ -2 & -5 & -1 & 0 & 1 & 0 \\ 2 & 4 & 0 & 0 & 0 & 1 \end{array} \right]$$

$R_2 + 2R_1$   
 $R_3 - 2R_1$

$$\left[ \begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & -2 & -4 & -2 & 0 & 1 \end{array} \right]$$

$R_1 - 3R_2$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & -7 & -5 & -3 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 2 & 2 & 2 & 1 \end{array} \right]$$

$R_3 + 2R_2$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & -7 & -5 & -3 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & \frac{1}{2} \end{array} \right]$$

$\frac{R_3}{2}$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 4 & \frac{7}{2} \\ 0 & 1 & 0 & -1 & -2 & -\frac{3}{2} \\ 0 & 0 & 1 & 1 & 1 & \frac{1}{2} \end{array} \right]$$

$R_1 + 7R_3$

$R_2 - 3R_3$

$\uparrow$   
I

$\uparrow$   
 $A^{-1}$

(23)

$$\left[ \begin{array}{ccc|ccc} 2 & 4 & 0 & 1 & 0 & 0 \\ 3 & 4 & -2 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array} \right]$$

$$\frac{R_1}{2} \left[ \begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{1}{2} & 0 & 0 \\ 3 & 4 & -2 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 + R_1 \end{array} \left[ \begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & -2 & -2 & -\frac{3}{2} & 1 & 0 \\ 0 & 3 & 2 & \frac{1}{2} & 0 & 1 \end{array} \right]$$

$$\frac{R_2}{(-2)} \left[ \begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 1 & \frac{3}{4} & -\frac{1}{2} & 0 \\ 0 & 3 & 2 & \frac{1}{2} & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} R_1 - 2R_2 \\ R_3 - 3R_2 \end{array} \left[ \begin{array}{ccc|ccc} 1 & 0 & -2 & -1 & 1 & 0 \\ 0 & 1 & 1 & \frac{3}{4} & -\frac{1}{2} & 0 \\ 0 & 0 & -1 & -\frac{7}{4} & \frac{3}{2} & 1 \end{array} \right]$$

$$\frac{1}{2} - 3\left(\frac{3}{4}\right) = \frac{2}{4} - \frac{9}{4} = -\frac{7}{4}$$

$$\frac{R_3}{(-1)} \left[ \begin{array}{ccc|ccc} 1 & 0 & -2 & -1 & 1 & 0 \\ 0 & 1 & 1 & \frac{3}{4} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{7}{4} & -\frac{3}{2} & -1 \end{array} \right]$$

$$\begin{array}{l} R_1 + 2R_3 \\ R_2 - R_3 \end{array} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{5}{4} & -2 & -2 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & \frac{7}{4} & -\frac{3}{2} & -1 \end{array} \right]$$

↑  
I

↑  
A<sup>-1</sup>

(35)

$$[A \mid I]$$

$$\left[ \begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array} \right]$$

$$R_2 - R_1 \quad \left[ \begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ \textcircled{0} & \textcircled{0} & -1 & 1 \end{array} \right]$$

A row of zeros on the left side indicates that  $A$  has no inverse.

(37)

$$\frac{1}{ad-bc} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{ad-bc} \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$