MATH 191 Review Problems

1. Evaluate $\lim_{x \to -8} \dfrac{x^2 + 5x - 24}{5x + 40}$.

2. Find $f'(x)$ using the limit definition of the derivative: $f(x) = \sqrt{2x+1}$.

3. Find $y'|_{x=2}$ for $y = (2x+1)^{\frac{2}{3}}(x^3 - 3x^2)$. Give an exact value.

4. Find $\dfrac{dy}{dx}$ for $y = \dfrac{8x^2+3}{5x+1}$. Simplify.

5. Find y' given $\cos(xy) - \sin(3y) = 1 + x^3$.

6. Find the equation of the tangent line to $y = \ln[x^3(x^2+4)]$ at $x = 1$. Write your answer in slope-intercept form.

7. We want to solve $e^x = \cos x + 1$. Use Newton's Method with $x_1 = -3$ to find x_2. Round your answer to 2 decimal places.

8. An object's position (in metres) is given by: $x = e^{-t^2+8t}, y = te^{7t}$. Find its velocity at $t = 0.2$ seconds. Round to 1 decimal place.

9. Water is stored in a cone-shaped container with height 14m and radius 5m. The water is dripping out of a small hole in the bottom at a rate of 2 m^3/h. At what rate is the water's depth changing when the depth is 6m?

10. For the function $f(x) = x^8 - 4x^6$:

    (a) Find all relative maximum or minimum points.

    (b) Find all points of inflection.

11. A rectangular box's length is two times its width. The width, length and height of the box add up to 140 cm. Find the maximum volume of the box.

12. Approximate $\sin(\frac{5\pi}{18})$ using linear approximation or differentials.

13. Find $f'(\frac{\pi}{6})$ for $f(x) = \csc^2(2x) + \tan^{-1}(5x)$. Round to 1 decimal place.

14. Find $f'(0)$ for $f(x) = \log_2(x^2 + 5x + 1) + 2^{4x}$. Simplify.

15. Integrate the following:

    (a) $\displaystyle\int 2x^2\sqrt{1-4x^3}\,dx$

    (b) $\displaystyle\int \frac{2x-1}{x^3}\,dx$

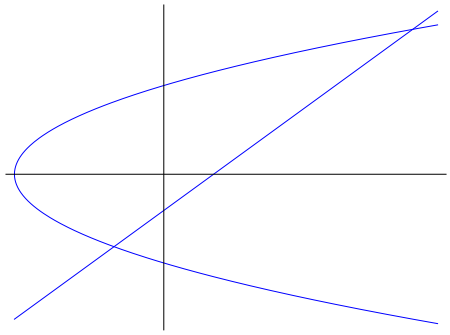
    (c) $\displaystyle\int_0^1 \frac{3x^4}{(1+7x^5)^2}\,dx$

    (d) $\displaystyle\int (3t^2+1)^2\,dt$

16. Use Simpson's Rule with $n = 4$ to approximate $\int_0^1 \sin x^3 \, dx$.

17. Find the displacement $s(t)$ of an object if its acceleration is given by $a = 12t$ m/s^2, its initial velocity is 5 m/s and its initial displacement is 0 m.

18. Find the area between the line $y = x - 1$ and the graph of $y^2 = 2x + 6$.



19. Find the volume of the solid of revolution generated by rotating the region bounded by $y = x^3$, $x = 0$, and $y = 8$ around the x-axis.

20. Find the centroid of the region bounded by $y = \sqrt{x}$, $x = 4$ and $y = 0$.

21. The cable of a bridge can be described by the equation $y = 0.04x^{3/2}$ from $x = 0$ to $x = 100$ m. Find the length of the cable.

22. (a) Find the inverse of $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & -4 \\ 3 & 0 & 1 \end{bmatrix}$

    (b) Use part (a) to solve the system below:

$$\begin{aligned} x \ + 2y - 3z &= -11 \\ 2x + 3y - 4z &= -14 \\ 3x \qquad + \ z \ &= \ 9 \end{aligned}$$

23. Solve the system below using Gauss-Jordan Elimination:

$$\begin{aligned} 2x + 8y - 10z &= -2 \\ 3x + 5y + \ 6z \ &= \ 4 \\ 4x + 2y + 22z &= 10 \end{aligned}$$

Answers:

1. $-\dfrac{11}{5}$
2. $\dfrac{1}{\sqrt{2x+1}}$
3. $-\dfrac{16}{3\sqrt[3]{5}}$
4. $\dfrac{40x^2+16x-15}{(5x+1)^2}$
5. $\dfrac{3x^2+y\sin(xy)}{-x\sin(xy)-3\cos(3y)}$
6. $y=\frac{17}{5}x+\ln 5-\frac{17}{5}$
7. -2.56
8. 37.5 m/s @ 15.1°
9. 0.14 m/h
10. (a) relative maximum (0,0), relative minima $(\pm\sqrt{3},-27)$
    (b) $(\pm\sqrt{\frac{15}{7}},-18.27)$
11. 90337.45 cm^3
12. 0.77
13. -2.4
14. $5\log_2 e+4\ln 2$ *or* $\dfrac{5}{\ln 2}+4\ln 2$
15. (a) $-\frac{1}{9}(1-4x^3)^{3/2}+C$
    (b) $-\frac{2}{x}+\frac{1}{2x^2}+C$
    (c) $\dfrac{3}{40}$
    (d) $\frac{9}{5}t^5+2t^3+t+C$
16. 0.2326
17. $2t^3+5t$
18. 18
19. $\dfrac{768}{7}\pi$
20. $\left(\dfrac{12}{5},\dfrac{3}{4}\right)$
21. 108.52 m
22. (a) $\begin{bmatrix} 3/2 & -1 & 1/2 \\ -7 & 5 & -1 \\ -9/2 & 3 & -1/2 \end{bmatrix}$
    (b) $x=2,\ y=-2,\ z=3$
23. $x=3-7t,\ y=3t-1,\ z=t$