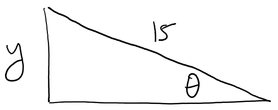
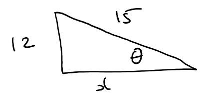
1




$$\sin\theta = \frac{y}{15}$$

Take $\frac{d}{dt}$: $\quad \cos\theta \frac{d\theta}{dt} = \frac{1}{15}\frac{dy}{dt}$

Solve for $\cos\theta$:




$$x = \sqrt{15^2 - 12^2}$$
$$= 9$$
$$\cos\theta = \frac{9}{15}$$

$$\frac{9}{15}\frac{d\theta}{dt} = \frac{1}{15}(2)$$

$$\frac{d\theta}{dt} = \frac{2}{9} \approx 0.22 \text{ rad/s}$$

2

$$u = 9x^2+1$$
$$du = 18x\,dx$$
$$\frac{du}{18} = x\,dx$$

$$\int \frac{x}{\sqrt{9x^2+1}}\,dx = \frac{1}{18}\int \frac{du}{\sqrt{u}}$$
$$= \frac{1}{18}\int u^{-1/2}\,du$$
$$= \frac{1}{18}\left(2u^{1/2}\right) + C$$
$$= \frac{1}{9}\left(9x^2+1\right)^{1/2} + C$$

3

$$u = 2 - x^3$$
$$du = -3x^2\,dx$$
$$-\frac{du}{3} = x^2\,dx$$

$$\int_0^1 x^2(2-x^3)^4\,dx = -\frac{1}{3}\int_{x=0}^{x=1} u^4\,du$$

$$= -\frac{1}{3}\,\frac{u^5}{5}\Bigg|_{x=0}^{x=1}$$

$$= -\frac{1}{15}(2-x^3)^5\Bigg|_0^1$$

$$= \frac{-1}{15} + \frac{32}{15}$$

$$= \frac{31}{15}$$

(4) $\frac{b-a}{n} = 0.5$ (size of step along $x$-axis)

$$\int_7^{10} f(x)\,dx \approx \frac{b-a}{3n}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6\right]$$

$$\approx \frac{0.5}{3}\left[98 + 4(112.5) + 2(128) + 4(144.5) + 2(162) + 4(180.5) + 200\right]$$

$$\approx 438$$

(5) Given: $a = -3t$ $\qquad v(0) = 12$

a) $v = \int -3t\,dt$

$v = \frac{-3t^2}{2} + C_1$

$\boxed{\text{Sub } v = 12,\ t = 0}$ : $12 = C_1$

$$\boxed{v = \frac{-3t^2}{2} + 12}$$

b) $s = \int\left(\frac{-3t^2}{2} + 12\right)dt$

$s = \frac{-t^3}{2} + 12t + C_2$

$\boxed{\text{Sub } s=0,\ t=0}$ : $0 = C_2$

$$\boxed{s = -\frac{t^3}{2} + 12t}$$

c) Solve $v = 0$

$$-\frac{3t^2}{2} + 12 = 0$$

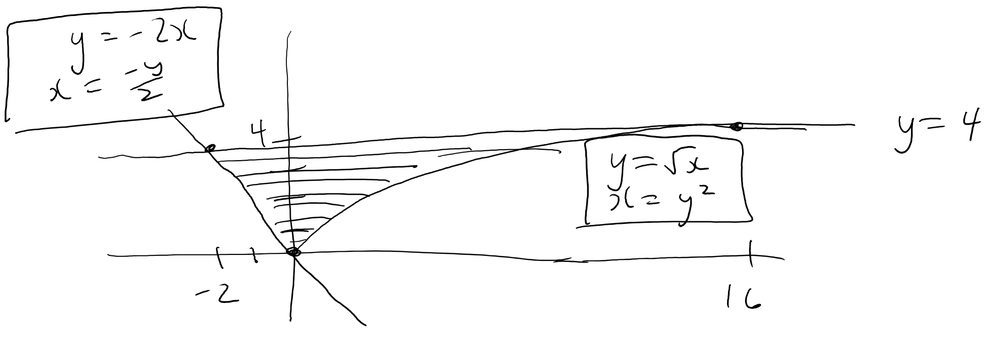
$$12 = \frac{3t^2}{2}$$

$$8 = t^2$$

$$t = \pm 2\sqrt{2}$$

$$t = 2\sqrt{2} \approx 2.8 \text{ seconds}$$

6




Horizontal slices

$$A = \int (x_r - x_l)\, dy$$

$$= \int_0^4 \left(y^2 + \frac{y}{2}\right) dy$$

$$= \frac{y^3}{3} + \frac{y^2}{4} \Big|_0^4$$

$$= \frac{64}{3} + 4$$

$$= \frac{76}{3}$$

Alternatively : Vertical Slices

$$A = \int_{-2}^{0} (4+2x)\,dx + \int_{0}^{16} (4-\sqrt{x})\,dx$$

$$\vdots$$

$$= \frac{76}{3}$$