# 27.8 Applications

**Ex:** Solve $e^x + \ln(x^2+5) - 4 = 0$
in the interval $[0,1]$

→ Newton's Method

$f(x) = e^x + \ln(x^2+5) - 4$

| $x$ | $f(x)$ |
|---|---|
| 0 | -1.39 |
| 1 | 0.51 |

closer to 0
Choose $x_0 = 1$

$f'(x) = e^x + \dfrac{2x}{x^2+5}$

| $x_n$ | $f(x_n)$ | $f'(x_n)$ | $x_n - \dfrac{f(x_n)}{f'(x_n)}$ | |
|---|---|---|---|---|
| $x_0 = 1$ | 0.5100 | 3.0516 | 0.8329 | (0.83) |
| $x_1 = 0.8329$ | 0.0393 | 2.5926 | 0.8177 | (0.82) |
| $x_2 = 0.8177$ | 0.0002 | 2.5538 | 0.8176 | (0.82) |

$x \approx 0.82$

Ex: A particle has position

$x = \log_2(3t+4)$ $\qquad$ $y = 3^{\sin t}$

Find its velocity at $t = 1$

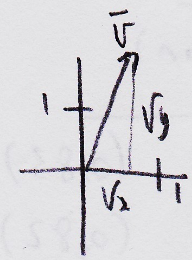
$$v_x = \frac{dx}{dt} = \frac{1}{\ln 2}\,\frac{3}{3t+4}$$

$$v_y = \ln 3 \cdot 3^{\sin t} \cdot \cos t$$

@ $t=1$ $\quad v_x = \dfrac{3}{7\ln 2}$ $\qquad v_y = \ln 3 \cdot 3^{\sin 1} \cos 1$

Radian Mode

$\simeq 0.6183$ $\qquad \simeq 1.4961$

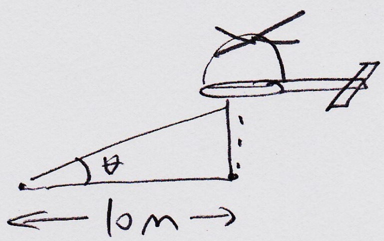


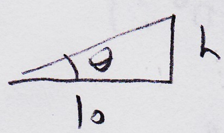

$$v = \sqrt{v_x^2 + v_y^2} \simeq 1.6 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) \quad (+180°?)$$
$$\simeq 68°$$

$$\boxed{1.6 \text{ m/s at } 68°}$$

Ex: Toy helicopter rises vertically at (a constant) 3 m/s. How fast is $\theta$ changing when height is 12m?




$$\frac{dh}{dt} = 3 \text{ m/s} \qquad \frac{d\theta}{dt} = ?$$
$$h = 12$$

$$\theta = \tan^{-1}\left(\frac{h}{10}\right)$$

$$\frac{d\theta}{dt} = \frac{d\theta}{dh}\frac{dh}{dt}$$

$$= \frac{1}{1+\left(\frac{h}{10}\right)^2} \cdot \frac{1}{10} \frac{dh}{dt}$$

$$= \frac{1}{1+\left(\frac{12}{10}\right)^2} \frac{1}{10} (3)$$

$$\approx 0.12 \text{ rad/s or } 7.04°/\text{s}$$

Ex: For what value of $\theta$ in $0 \le \theta < 2\pi$ is $f(\theta) = 5\sin\theta + 6\cos\theta$ maximized?

$\rightarrow$ Set $f'(\theta) = 0$

$$f'(\theta) = 5\cos\theta - 6\sin\theta$$

$$5\cos\theta - 6\sin\theta = 0$$

$$5\cos\theta = 6\sin\theta$$

$$\frac{5}{6} = \tan\theta$$

$$\theta = \tan^{-1}\left(\tfrac{5}{6}\right) \quad (+180°?)$$

$\theta \simeq 39.8°$ $\qquad$ $\theta \approx 219.8°$

$$f''(\theta) = -5\sin\theta - 6\cos\theta$$

$f''(39.8°) < 0$ $\qquad$ $f''(219.8°) > 0$

$\cap$ CD $\qquad$ $\cup$ CU

$\Rightarrow$ rel max $\qquad$ $\Rightarrow$ rel min

Use $\theta \simeq 39.8°$

Exact value: $\theta = \tan^{-1}\left(\frac{5}{6}\right)$

Ex: Use a linear approximation to estimate $\sin\left(\frac{17\pi}{96}\right)$

$$f(x) \simeq f(a) + f'(a)(x-a)$$

a: near $\frac{17\pi}{96}$

$\sin a$ is known

Use $a = \frac{16\pi}{96} = \frac{\pi}{6}$

Sub $f(x) = \sin x$ $\qquad a = \frac{\pi}{6}$

$f'(x) = \cos x$

$$\sin x \simeq \sin\frac{\pi}{6} + \cos\frac{\pi}{6}\left(x - \frac{\pi}{6}\right)$$

$$\sin\frac{17\pi}{96} \simeq \sin\frac{\pi}{6} + \cos\frac{\pi}{6}\left(\frac{17\pi}{96} - \cancel{\frac{\pi}{6}}\right)$$

$\frac{16\pi}{96}$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{\pi}{96}\right)$$

$$= \frac{1}{2} + \frac{\sqrt{3}\,\pi}{192}$$