27.6 Derivatives of Exponential Functions

$$\frac{d}{dx}[b^x] = \ln b \cdot b^x$$

b: constant

Ex: Find $f'(x)$

a) $f(x) = e^x$

$f'(x) = e^x$ $\quad (\ln e = 1)$

b) $f(x) = 3^x$

$f'(x) = \ln 3 \cdot 3^x$

Chain Rule for Exponential Functions

$$\boxed{\frac{d}{dx}[b^u] = \ln b \cdot b^u \cdot \frac{du}{dx}}$$

b: constant

e^x is sometimes written $\exp(x)$

e^{x^2} " $\exp(x^2)$

outside inside

Ex: a) $\frac{d}{dx}[e^{x^2}] = \cancel{\ln e}\ e^{x^2} \cdot 2x$

$= 2x e^{x^2}$

b) $\frac{d}{dx}\left[2^{\sqrt{x}}\right] = \ln 2 \cdot 2^{\sqrt{x}} \frac{d}{dx}\left[x^{1/2}\right]$

$= \ln 2 \cdot 2^{\sqrt{x}} \frac{1}{2} x^{-1/2}$

$= \dfrac{\ln 2 \cdot 2^{\sqrt{x}}}{2\sqrt{x}}$

c) $\frac{d}{dx}\left[e^{x+7}\right] = \cancel{\ln e}\; e^{x+7} (1)$

$= e^{x+7}$

d) $\frac{d}{dx}\left[x^3 e^{\sqrt{x}}\right] = x^3\left(\cancel{\ln e}\; e^{\sqrt{x}} \frac{1}{2} x^{-1/2}\right) + e^{\sqrt{x}}(3x^2)$

$= \frac{1}{2} x^{5/2} e^{\sqrt{x}} + 3x^2 e^{\sqrt{x}}$

$= \frac{1}{2} e^{\sqrt{x}}\left(x^{5/2} + 6x^2\right)$

Ex: Find $f'(x)$

a) $f(x) = 9e^{2x}(e^{3x} + e^{4x})$ expand

$f(x) = 9(e^{5x} + e^{6x})$

$f'(x) = 9(e^{5x} \cdot 5 + e^{6x} \cdot 6)$

$= 9(5e^{5x} + 6e^{6x})$

b) $f(x) = \dfrac{e^{3x} + e^{4x}}{e^{2x}}$

$= e^{-2x}(e^{3x} + e^{4x})$

$= e^{x} + e^{2x}$

$f'(x) = e^{x} + 2e^{2x}$

c) $f(x) = (4e^{7x})^2 e^{6x}$

$= 16e^{14x} e^{6x}$

$= 16e^{20x}$

$f'(x) = 16e^{20x} \cdot 20$

$= 320e^{20x}$