27.5 Review: Logarithms

$\log_2 1 = 0$ ← exponent $\quad 2^? = 1$

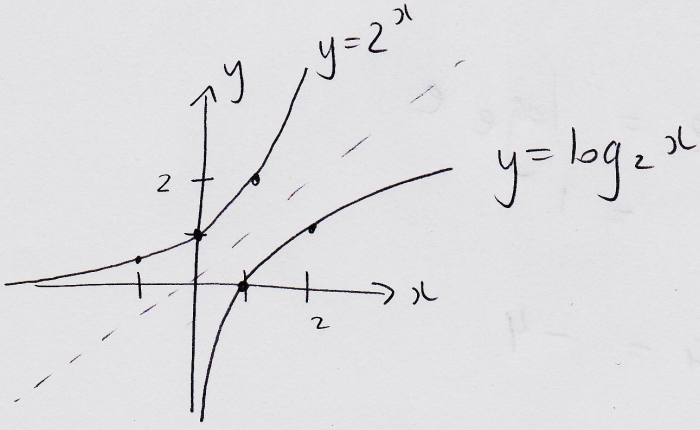
$\log_2 2 = 1$

$\log_2 8 = 3$

$\log_2 \frac{1}{2} = -1$

$\log_2 \frac{1}{16} = -4$

$\log_2 2^7 = 7$



Euler's number $e \approx 2.718$

Notation: "$\log x$" means $\log_{10} x$

"$\ln x$" means $\log_e x$

Ex: $\log 10 = \log_{10} 10$
$= 1$

$\log 0.1 = -1$

$\ln e = \log_e e$
$= 1$

$\ln \frac{1}{e^4} = -4$

Change of Base Formula

$$\log_a b = \frac{\ln b}{\ln a}$$

Ex: Calculate $\log_2 5$

$$= \frac{\ln 5}{\ln 2}$$

$$\approx 2.32$$

Ex: Simplify

a) $\log e$

$$= \log_{10} e$$

$$= \frac{\ln e}{\ln 10}$$

$$= \frac{1}{\ln 10}$$

b) $\log_b e$

$$= \frac{\ln e}{\ln b}$$

$$= \frac{1}{\ln b}$$

Log Rules

$$\log_b(xy) = \log_b x + \log_b y$$

any base

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b x^n = n \log_b x$$

Ex: Simplify $\ln \sqrt{x}$

$$= \ln x^{1/2}$$

$$= \frac{1}{2} \ln x$$

Ex: Simplify $\ln\left[\dfrac{\sin(2x)\sqrt{x^2+6}}{(x+1)^4}\right]$

$$= \ln\left[\sin(2x)\,(x^2+6)^{1/2}\right] - \ln(x+1)^4$$

$$= \ln(\sin(2x)) + \ln(x^2+6)^{1/2} - \ln(x+1)^4$$

$$= \ln(\sin(2x)) + \frac{1}{2}\ln(x^2+6) - 4\ln(x+1)$$

27.5 Derivatives of Logarithmic Functions

$$\frac{d}{dx}[\log_b x] = \frac{1}{\ln b} \cdot \frac{1}{x} \quad \text{or} \quad \log_b e \cdot \frac{1}{x}$$

$$\log_b e = \frac{\ln e}{\ln b} = \frac{1}{\ln b}$$

Special Case: $\frac{d}{dx}[\ln x] = \frac{1}{x}$

Why? $\frac{d}{dx}[\ln x] = \frac{d}{dx}[\log_e x] = \frac{1}{\ln e} \cdot \frac{1}{x} = \frac{1}{x}$

Ex: Find $f'(x)$

a) $f(x) = \ln x$

$f'(x) = \frac{1}{x}$

b) $f(x) = \ln(4x^2+1)$

$f'(x) = \frac{1}{4x^2+1} \cdot 8x$ Chain Rule

c) $f(x) = \log_2 x$

$f'(x) = \frac{1}{\ln 2} \cdot \frac{1}{x}$ or $\frac{\log_2 e}{x}$

d) $f(x) = \log_3 (7x+1)$

$f'(x) = \frac{1}{\ln 3} \cdot \frac{1}{7x+1} \cdot 7$ or $\frac{7(\log_3 e)}{7x+1}$

e) $f(x) = \log (x^2+2)$

$= \log_{10} (x^2+2)$

$f'(x) = \frac{1}{\ln 10} \cdot \frac{1}{x^2+2} \cdot 2x$ or $\frac{2x \log_{10} e}{x^2+2}$ or $\frac{2x \log e}{x^2+2}$

f) $f(x) = \ln (\sin x^2)$

$= \frac{1}{\sin x^2} \cdot \frac{d}{dx}[\sin x^2]$

$= \frac{\cos x^2 \cdot 2x}{\sin x^2}$ or $2x \cot x^2$

Ex: $f(x) = \ln^4(x^3+17x+1)$ Find $f'(x)$

$$f(x) = [\ln(x^3+17x+1)]^4$$

$$f'(x) = 4[\ln(x^3+17x+1)]^3 \cdot \frac{1}{x^3+17x+1} \cdot (3x^2+17)$$

$$= \frac{4(3x^2+17)}{x^3+17x+1} \ln^3(x^3+17x+1)$$

Ex: $f(x) = \ln \frac{(4x+1)^2}{(5x+3)}$ Find $f'(x)$

→ Use log rules first

$$f(x) = \ln(4x+1)^2 - \ln(5x+3)$$

$$f(x) = 2\ln(4x+1) - \ln(5x+3)$$

$$f'(2) = 2 \cdot \frac{1}{4x+1} \cdot 4 - \frac{1}{5x+3} \cdot 5$$

$$= \frac{8}{4x+1} - \frac{5}{5x+3}$$

or $$\frac{8(5x+3) - 5(4x+1)}{(4x+1)(5x+3)} = \frac{20x+19}{(4x+1)(5x+3)}$$