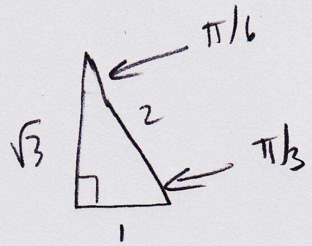
27.3 Review: Inverse Trig Functions

$\sin(\text{angle}) = \#$

$\arcsin(\#) = \text{angle}$

$\arcsin x = \sin^{-1} x$

not to be confused with $(\sin x)^{-1} = \dfrac{1}{\sin x} = \csc x$

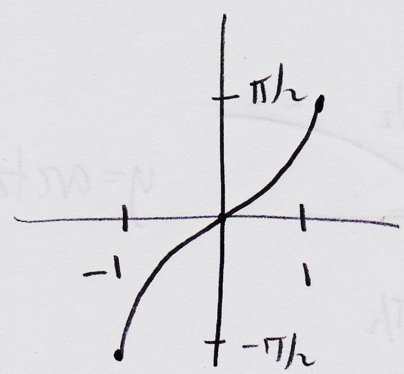


$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

$$\arcsin \frac{1}{2} = \frac{\pi}{6}$$

$$\boxed{\sin^{-1}(-a) = -\sin^{-1} a}$$

$$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$



$y = \arcsin x$

$$-1 \le x \le 1$$

$$-\frac{\pi}{2} \le \arcsin x \le \frac{\pi}{2}$$

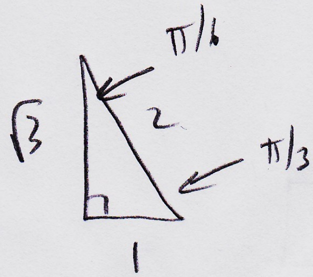
$\tan(\text{angle}) = \#$

$\arctan(\#) = \text{angle}$

$\arctan x = \tan^{-1} x$

not to be confused with

$$(\tan x)^{-1} = \frac{1}{\tan x} = \cot x$$

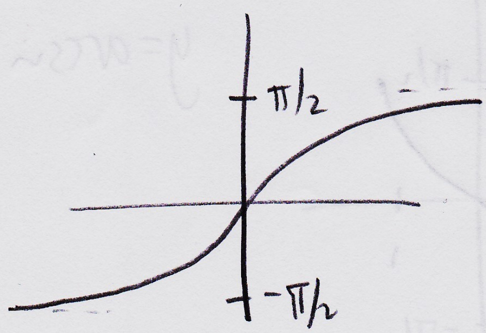


$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$\arctan \sqrt{3} = \frac{\pi}{3}$$

$$\boxed{\arctan(-a) = -\arctan(a)}$$

$$\arctan(-1) = -\frac{\pi}{4}$$



$y = \arctan x$

$$-\infty < x < \infty$$

$$-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$$

$\underline{Ex}$: $f(x) = \sqrt{4 - \boxed{\sin^{-1}(3x-8)}}$

Find $f(2.8)$

Want #, not degrees

$\boxed{\text{Radian Mode}}$

$f(2.8) = \sqrt{4 - \sin^{-1}(0.4)}$

≈ 1.9

$\underline{Ex}$: $f(x) = \cos^{-1}\left(\dfrac{\cos 2x}{x+1}\right)$

Find $f\left(\frac{\pi}{7}\right)$

$\boxed{\text{Radian Mode}}$

$f\left(\frac{\pi}{7}\right) = \cos^{-1}\left(\dfrac{\cos\left(\frac{2\pi}{7}\right)}{\frac{\pi}{7}+1}\right)$

≈ 1.1 rad or $64.5°$

$\left(\times \dfrac{180°}{\pi}\right)$

Applications mostly use

arcsin x, arctan x

27.3 Derivatives of Inverse Trig Functions

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$$

<u>Ex</u>: Find $f'(x)$

a) $f(x) = \sin^{-1}(7x)$

$$f'(x) = \frac{1}{\sqrt{1-(7x)^2}} \cdot 7 \quad \text{Chain Rule}$$

$$= \frac{7}{\sqrt{1-49x^2}}$$

b) $f(x) = \frac{1}{3}\sin^{-1}\left(\frac{x}{4}\right)$

$$f'(x) = \frac{1}{3} \cdot \frac{1}{\sqrt{1-\left(\frac{x}{4}\right)^2}} \cdot \frac{1}{4}$$

$$= \frac{1}{3\sqrt{1-\left(\frac{x^2}{16}\right)} \cdot \sqrt{16}}$$

$$= \frac{1}{3\sqrt{16-x^2}}$$

c) $f(x) = x\cos^{-1}x^2$

$$f'(x) = x \cdot \frac{-1}{\sqrt{1-(x^2)^2}} \cdot 2x + \cos^{-1}x^2$$

$$= \frac{-2x^2}{\sqrt{1-x^4}} + \cos^{-1}x^2$$

d) $f(x) = \tan^{-1}2x$

$$f'(x) = \frac{1}{1+(2x)^2} \cdot 2$$

$$= \frac{2}{1+4x^2}$$

e) $f(x) = \cos^{-1} \sqrt[3]{x}$

$= \cos^{-1} x^{1/3}$

$$f'(x) = \frac{-1}{\sqrt{1-(x^{1/3})^2}} \cdot \frac{1}{3} x^{-2/3}$$

$$= \frac{-1}{3x^{2/3}\sqrt{1-x^{2/3}}}$$

f) $f(x) = (\tan^{-1} 8x - 5x^2)^3$

$$f'(x) = 3(\tan^{-1} 8x - 5x^2)^2 \left[\frac{1}{1+(8x)^2} \cdot 8 - 10x\right]$$

$$= 3(\tan^{-1} 8x - 5x^2)^2 \left[\frac{8}{1+64x^2} - 10x\right]$$

or $\dfrac{8 - 10x - 640x^3}{1+64x^2}$

or $$\frac{6(\tan^{-1} 8x - 5x^2)^2 (4 - 5x - 320x^3)}{1+64x^2}$$

Ex: $y = \dfrac{\tan^{-1}x}{1+x^2}$

Find $\left.\dfrac{dy}{dx}\right|_{x=-1}$

$$\frac{dy}{dx} = \frac{(1+x^2)\frac{1}{1+x^2} - \tan^{-1}x\,(2x)}{(1+x^2)^2}$$

$$= \frac{1-2x\tan^{-1}x}{(1+x^2)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=-1} = \frac{1+2\tan^{-1}(-1)}{2^2}$$

$$= \frac{1+2\left(\frac{-\pi}{4}\right)}{4}\cdot\frac{2}{2}$$

$$= \frac{2-\pi}{8}$$

Ex: Find $\frac{d}{dx} \tan^{-1}\left(\frac{k}{x}\right)$ k: constant

$$= \frac{1}{1+\left(\frac{k}{x}\right)^2} \cdot \frac{d}{dx}\left(kx^{-1}\right]$$

$$= \frac{1}{1+\frac{k^2}{x^2}} \cancel{\left(-kx^{-2}\right)} \cdot \frac{-k}{x^2}$$

$$= \frac{-k}{x^2+k^2}$$