

27.2 Derivatives of Other Trig Functions

$$\frac{d}{dx} [\tan x] = \sec^2 x$$

$$\frac{d}{dx} [\sec x] = \sec x \tan x$$

$$\frac{d}{dx} [\cot x] = -\csc^2 x$$

$$\frac{d}{dx} [\csc x] = -\csc x \cot x$$

Ex: Find $f'(x)$

a) $f(x) = \tan 9x$

$$\begin{aligned} f'(x) &= \sec^2 9x \cdot 9 \\ &= 9\sec^2 9x \end{aligned}$$

b) $f(x) = \sec 8x$

$$\begin{aligned} f'(x) &= \sec 8x \tan 8x \cdot 8 \\ &= 8\sec 8x \tan 8x \end{aligned}$$

$$c) f(x) = 2 \csc(1 + 3x)$$

$$\begin{aligned} f'(x) &= -2 \csc(1 + 3x) \cot(1 + 3x) \cdot 3 \\ &= -6 \csc(1 + 3x) \cot(1 + 3x) \end{aligned}$$

$$d) f(x) = \sqrt{3x + \tan 2x}$$

$$\begin{aligned} f'(x) &= \frac{1}{2} (3x + \tan 2x)^{-1/2} \cdot (3 + \sec^2 2x \cdot 2) \\ &= \frac{3 + 2 \sec^2 2x}{2 \sqrt{3x + \tan 2x}} \end{aligned}$$

$$e) f(x) = 3 \cot^3 \frac{x}{9}$$

$$= 3 \left[\cot \frac{x}{9} \right]^3$$

$$f'(x) = 9 \left[\cot \frac{x}{9} \right]^2 \left(-\csc^2 \frac{x}{9} \left(\frac{1}{9} \right) \right)$$

$$= -\cot^2 \frac{x}{9} \csc^2 \frac{x}{9}$$

$$f) f(x) = 7 \sin x \sec^2 x^2$$

$$\begin{aligned} f'(x) &= 7 \sin x \frac{d}{dx} [\sec^2 x^2] + \sec^2 x^2 (7 \cos x) \\ &= 7 \sin x (\sec^2 x^2 \tan x^2 \cdot 2x) + 7 \cos x \sec^2 x^2 \end{aligned}$$

$$\text{or } 7 \sec^2 x^2 (2x \sin x \tan x^2 + \cos x)$$