27.1 Derivatives of Sine and Cosine

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

Recall Chain Rule

$$\frac{d}{dx}(1+4x)^2 = 2(1+4x)\cdot 4$$

$$\frac{d}{dx}\sin x^2 = \cos x^2 \cdot 2x$$
$$= 2x\cos x^2$$

$$\frac{d}{dx}\sin^2 x = \frac{d}{dx}[\sin x]^2$$
$$= 2\sin x \cos x$$

Ex: Find $f'(x)$

a) $f(x) = 4\sin(7x+2)$

$f'(x) = 4\cos(7x+2)\cdot 7$

$= 28\cos(7x+6)$

b) $f(x) = \cos^4 x$

$f(x) = [\cos x]^4$

$f'(x) = 4(\cos x)^3(-\sin x)$

$= -4\cos^3 x \sin x$

c) $f(x) = \cos^2\left(1 - \frac{\pi x}{2}\right)$

$f(x) = \left[\cos\left(1 - \frac{\pi x}{2}\right)\right]^2$

$f'(x) = 2\cos\left(1 - \frac{\pi x}{2}\right) \frac{d}{dx} \cos\left(1 - \frac{\pi x}{2}\right)$

$= 2\cos\left(1 - \frac{\pi x}{2}\right)\left[-\sin\left(1 - \frac{\pi x}{2}\right)\right]\left(\frac{-\pi}{2}\right)$

$= \pi\cos\left(1 - \frac{\pi x}{2}\right)\sin\left(1 - \frac{\pi x}{2}\right)$

d) $f(x) = x\cos 7x$

$$f'(x) = x(-\sin 7x \cdot 7) + \cos 7x \cdot 1$$

$$= -7x\sin 7x + \cos 7x$$

e) $f(x) = \dfrac{\sin x}{1+x^2}$

$$f'(x) = \frac{(1+x^2)\cos x - \sin x(2x)}{(1+x^2)^2}$$

$$= \frac{\cos x + x^2\cos x - 2x\sin x}{(1+x^2)^2}$$

<u>Ex</u>: Find $\dfrac{dy}{dx}$ $\quad \sin(xy) + \cos 3y = x^3$

$$\cos(xy)\frac{d}{dx}(xy) - 3\sin 3y\frac{dy}{dx} = 3x^2$$

$$\cos(xy)\left[x\frac{dy}{dx} + y\right] - 3\sin 3y\frac{dy}{dx} = 3x^2$$

$$x\cos(xy)\frac{dy}{dx} + y\cos(xy) - 3\sin 3y\frac{dy}{dx} = 3x^2$$

$$\left[x\cos(xy) - 3\sin(3y)\right]\frac{dy}{dx} = 3x^2 - y\cos(xy)$$

$$\frac{dy}{dx} = \frac{3x^2 - y\cos(xy)}{x\cos(xy) - 3\sin(3y)}$$