26.1 Applications of Integration

Recall $s(t)$ or $h(t)$ = displacement

$v(t)$ = velocity

$a(t)$ = acceleration

take derivative ↓ ↑ integrate

$$\boxed{\begin{aligned} v(t) &= s'(t) \\ a(t) &= v'(t) = s''(t) \end{aligned}}$$

$v(t) = \int a(t)\,dt$

$s(t) = \int v(t)\,dt$

Ex: A ball is thrown straight up from the ground with initial velocity 3 m/s. Find height $h(t)$.

$a(t) = -9.8$ (gravity)

$v(t) = \int a(t)\,dt$

$v(t) = \int -9.8\,dt$

$\boxed{v(t) = -9.8t + C}$

$v(0)=3$: $3 = C$

$\boxed{v(t) = -9.8t + 3}$

$h(t) = \int v(t)\,dt$

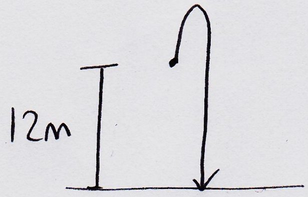
$h(t) = \int (-9.8t + 3)\,dt$

$\boxed{h(t) = -4.9t^2 + 3t + C_1}$ new name

$h(0) = 0: \quad 0 = C_1$

$h(t) = -4.9t^2 + 3t$

<u>Ex</u>:



Ball is thrown straight up from 12m high. Takes 6s to land. Find $h(t)$ and initial velocity v_0

$a(t) = -9.8 \quad$ (gravity)

$v(t) = \int -9.8\,dt$

$\boxed{v(t) = -9.8t + C_1}$

$v(0) = v_0: \quad v_0 = C_1$

$\boxed{v(t) = -9.8t + v_0}$

$h(t) = \int (-9.8t + v_0)\,dt$

$\boxed{h(t) = -4.9t^2 + v_0 t + C_2}$

$h(0)=12$: $\quad 12 = C_2$

$$\boxed{h(t) = -4.9t^2 + v_0 t + 12}$$

Given $h(6)=0$: $\quad 0 = -4.9(36) + 6v_0 + 12$

$$v_0 = 27.4 \text{ m/s}$$

Ex: Car travels in straight line with $a(t) = -4t$ m/s² (slowing down). Brakes are applied when velocity = 20 m/s. Stopping distance?

$$\boxed{\begin{array}{l} t=0: \text{ brakes applied} \\ v(0) = 20 \text{ m/s} \\ s(0) = 0 \end{array}}$$

1) Find $v(t)$, $s(t)$

$$a(t) = -4t$$

$$v(t) = \int -4t\,dt$$

$$= -2t^2 + C_1$$

$v(0)=20$: $\quad 20 = C_1$

$$\boxed{v(t) = -2t^2 + 20}$$

$$s(t) = \int (-2t^2 + 20)\,dt$$

$$= -\frac{2t^3}{3} + 20t + C_2$$ ⟸ new name

$s(0)=0$: $0 = C_2$

$$\boxed{s(t) = \frac{-2t^3}{3} + 20t}$$

2) Stopping time?

Set $v(t) = 0$

$$-2t^2 + 20 = 0$$

$$t^2 = 10$$

$$t = \pm\sqrt{10}$$

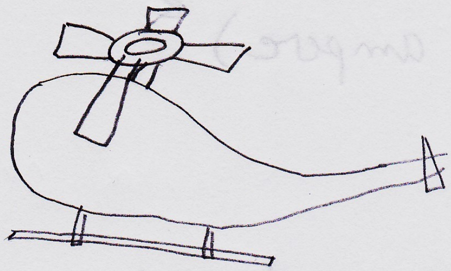
$$t = \sqrt{10}$$

3) Stopping distance

$$s(\sqrt{10}) = \frac{-2\sqrt{10}^3}{3} + 20\sqrt{10}$$

$$\approx 42.2\,m$$

Ex:



Angular velocity of rotor is $\frac{d\theta}{dt} = \sqrt{(4t+1)^3}$

Find angular displacement $\theta(t)$ after 5s.

$$\theta = \int \frac{d\theta}{dt}\, dt$$

$$= \int (4t+1)^{3/2} dt$$

$u = 4t+1$

$du = 4dt$

$\frac{du}{4} = dt$

$$= \frac{1}{4}\int u^{3/2} du$$

$$= \frac{1}{4}\left(\frac{2}{5}u^{5/2}\right) + C$$

$$\boxed{\theta = \frac{1}{10}(4t+1)^{5/2} + C}$$

$\theta(0)=0$:

$$0 = \frac{1}{10} + C$$

$$C = \frac{-1}{10}$$

$$\theta = \frac{1}{10}(4t+1)^{5/2} - \frac{1}{10}$$

$$\theta(5) = \frac{1}{10}(21)^{5/2} - \frac{1}{10}$$

$$\approx 201.99 \text{ rad}$$

$(\div 2\pi)$ or ≈ 32 full rotations