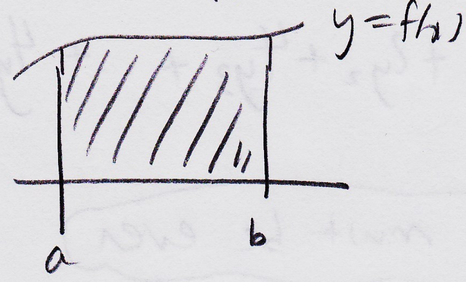
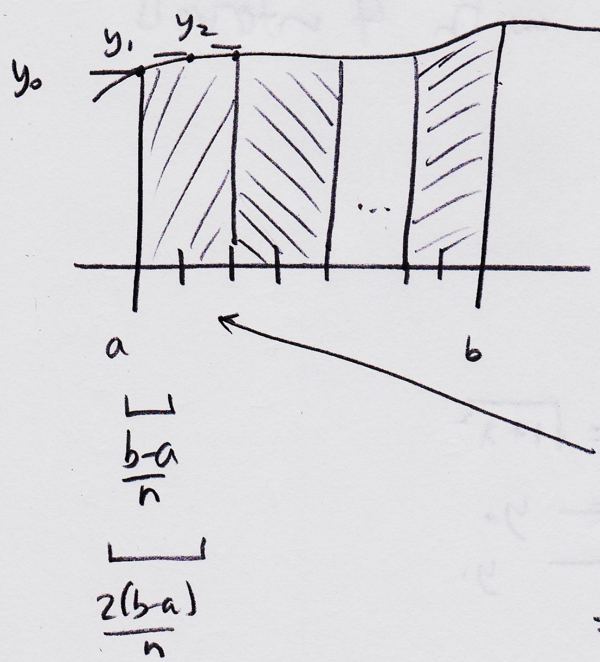
25.6 Simpson's Rule

Want to approximate $\int_a^b f(x)\,dx$



Divide into n intervals
n must be even



approx. area

= base (approx. height)

$= \frac{2(b-a)}{n} \left[\frac{y_0 + 4y_1 + y_2}{6} \right]$

weighted average of heights

$= \frac{b-a}{3n} \left[y_0 + 4y_1 + y_2 \right]$

Total approx. area $= \frac{b-a}{3n}\left[y_0 + 4y_1 + y_2 + y_2 + 4y_3 + y_4 + \dots\right]$

$$= \frac{b-a}{3n}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + \dots + 4y_{n-1} + y_n\right]$$

Simpson's Rule

n must be even

Ex: Approximate $\int_1^5 \sqrt{1+x^2}\,dx$ using Simpson's Rule with 4 intervals. Answer to 2 d.p.

$n=4$ even ✓

$\frac{b-a}{n} = \frac{5-1}{4} = 1$

x	$y = \sqrt{1+x^2}$	
1	$\sqrt{2}$	$\leftarrow y_0$
2	$\sqrt{5}$	$\leftarrow y_1$
3	$\sqrt{10}$	
4	$\sqrt{17}$	
5	$\sqrt{26}$	$\leftarrow y_4$

start at $a=1$
end at $b=5$
go up by $\frac{b-a}{n} = 1$

$$\int_1^5 \sqrt{1+x^2}\,dx \simeq \frac{b-a}{3n}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$\simeq \frac{1}{3}\left[\sqrt{2} + 4\sqrt{5} + 2\sqrt{10} + 4\sqrt{17} + \sqrt{26}\right]$$

$$\simeq 12.76$$

Ex: Approximate $\int_0^2 7^x\,dx$ to 2 d.p. using

a) Trapezoidal Rule with $n=4$

b) Simpson's "

a) $\frac{b-a}{n} = \frac{2-0}{4} = 0.5$

x	$y = 7^x$
0	1
0.5	$7^{0.5}$
1	7
1.5	$7^{1.5}$
2	49

$$\int_0^2 7^x\,dx \simeq \frac{b-a}{2n}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right]$$

$\frac{0.5}{2}$

$$\simeq 26.58$$

b) $n=4$ is even ✓

$\frac{b-a}{n} = 0.5$

Use y-values above

$$\int_0^2 7^x \, dx \simeq \frac{b-a}{3n}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$\simeq \frac{0.5}{3}\left[1 + 4(7^{0.5}) + 2(7) + 4(7^{1.5}) + 49\right]$$

$$\simeq 24.78$$

For reference: true value $\simeq 24.67$

Simpson's Rule is generally better