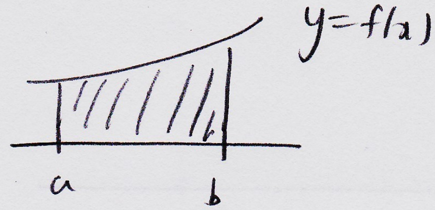
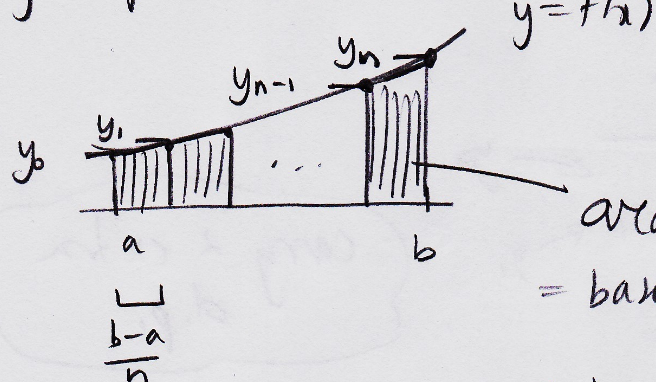
25.5 The Trapezoidal Rule

Want to approximate $\int_a^b f(x)\,dx$



Divide into n intervals and approximate by trapezoids



area of trapezoid

$$= \text{base}\left(\frac{\text{height}_1 + \text{height}_2}{2}\right)$$

$$= \frac{b-a}{n}\left(\frac{y_{n-1}+y_n}{2}\right)$$

$$= \frac{b-a}{2n}\left(y_{n-1}+y_n\right)$$

$$\text{Approx. area} = \frac{b-a}{2n}(y_0+y_1)+\dots+\frac{b-a}{2n}(y_{n-1}+y_n)$$

$$= \frac{b-a}{2n}\left[y_0+y_1+y_1+y_2+\dots+y_{n-1}+y_n\right]$$

$$= \frac{b-a}{2n}\left[y_0+2y_1+\dots+2y_{n-1}+y_n\right]$$

Trapezoidal Rule

Ex: Approximate using trapezoidal rule with 4 intervals. Answer to 4 decimal places.

$$\int_2^3 \frac{1}{x}\,dx$$

$$\frac{b-a}{n} = \frac{3-2}{4} = \frac{1}{4}$$

x	$y = \frac{1}{x}$	
2	0.5	$\leftarrow y_0$
2.25	0.444444	$\leftarrow y_1$
2.5	0.4	
2.75	0.363636	
3	0.333333	$\leftarrow y_4$

carry 2 extra d.p.

Start at $a = 2$
End at $b = 3$
go up by $\frac{b-a}{n} = 0.25$

$$\int_2^3 \frac{1}{x}\,dx \approx \underbrace{\frac{b-a}{2n}}_{\frac{0.25}{2}}\left[y_0 + 2y_1 + \cdots + 2y_3 + y_4\right)$$

$$\approx 0.4062$$

As # intervals $\to \infty$,

approximation $\to$ true value of integral

Ex: Approximate using trapezoidal rule with 5 intervals. Answer to 2 d.p.

$$\int_0^2 3^x \, dx$$

$$\frac{b-a}{n} = \frac{2-0}{5} = 0.4$$

x	$y = 3^x$	
0	1	$\leftarrow y_0$
0.4	$3^{0.4}$	$\leftarrow y_1$
0.8	$3^{0.8}$	
1.2	$3^{1.2}$	
1.6	$3^{1.6}$	
2	9	$\leftarrow y_5$

$\frac{0.4}{2}$

$$\int_0^2 3^x \, dx \simeq \frac{b-a}{2n}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + 2y_4 + y_5\right]$$

$$\simeq 7.40$$