25.4 The Definite Integral

<u>Recap</u>: $$\int_2^4 (x^3+1)\,dx = \left[\frac{x^4}{4} + x\right]_2^4$$

$$= \left(\frac{4^4}{4} + 4\right) - \left(\frac{2^4}{4} + 2\right)$$

$$= 62$$

"definite integral"

$$\int (x^3+1)\,dx = \frac{x^4}{4} + x + C$$

"indefinite integral"

<u>Ex</u>: Evaluate $\int_0^1 3x^2(x^3+2)^4\,dx$

$u = x^3 + 2$

$du = 3x^2\,dx$

when $x = 0$, $u = 2$

$x = 1$, $u = 3$

$$\int = \int_2^3 u^4\,du$$

$$= \left[\frac{u^5}{5}\right]_2^3$$

$= \frac{3^5}{5} - \frac{2^5}{5}$

$= \frac{211}{5}$

Ex: Evaluate $\int_0^1 (x^2+2x^5)\sqrt{x^3+x^6}\,dx$

$u = x^3 + x^6$

$du = (3x^2+6x^5)\,dx$

$\frac{du}{3} = (x^2+2x^5)\,dx$

when $x=0$, $u=0$

$x=1$, $u=2$

$\int = \int_0^2 \sqrt{u}\,\frac{du}{3}$

$= \frac{1}{3}\int_0^2 u^{1/2}\,du$

$= \frac{1}{3}\left[\frac{2}{3}u^{3/2}\right]_0^2$

$= \frac{2}{9}\cdot 2^{3/2} - 0$

$= \frac{2^{5/2}}{9}$

$\underline{Ex}$: Evaluate $\displaystyle\int_1^2 \frac{dx}{\sqrt{6x+1}}$

$u = 6x+1$

$du = 6\,dx$

$\frac{du}{6} = dx$

when $x=1$, $u=7$

$x=2$, $u=13$

$$I = \int_7^{13} \frac{du}{6\sqrt{u}}$$

$$= \frac{1}{6}\int_7^{13} u^{-1/2}\,du$$

$$= \frac{1}{6}\left[2u^{1/2}\right]_7^{13}$$

$$= \frac{1}{3}\left(\sqrt{13}-\sqrt{7}\right)$$

<u>Ex</u>: Evaluate $\int_{-1}^{0} x(x+1)^6\,dx$

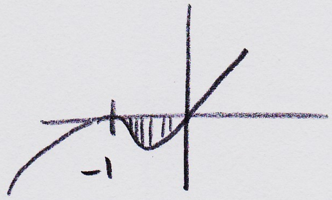
$u = x+1$
$du = dx$

$x = ?$
$x = u-1$

when $x=-1$, $u=0$
$x=0$, $u=1$

$$\int = \int_0^1 (u-1)u^6\,du$$
$$= \int_0^1 (u^7 - u^6)\,du$$
$$= \left[\frac{u^8}{8} - \frac{u^7}{7}\right]_0^1$$
$$= \frac{1}{8} - \frac{1}{7}$$
$$= -\frac{1}{56}$$

Integrals can be negative if function is below x-axis



$y = x(x+1)^6$

Ex: Evaluate $\int_0^1 (x+3)(2-x)\,dx$

Expand! $= \int_0^1 (\cancel{2x - x^2 + 6 - 3x})\,dx$

$6 - x - x^2$

$= \left[6x - \frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= \left(6 - \frac{1}{2} - \frac{1}{3}\right) - 0$

$= \frac{31}{6}$

Ex: Evaluate $\int_2^4 \sqrt{x^2+2x+1}\,dx$

Simplify!

$= \int_2^4 \sqrt{(x+1)^2}\,dx$

Recall $\sqrt{a^2} = |a|$

$\sqrt{(x+1)^2} = x+1$ provided $x+1 \geq 0$

Here $2 \leq x \leq 4$

So $x+1 \geq 0$ ✓

$= \int_2^4 (x+1)\,dx$

$= \left[\frac{x^2}{2} + x\right]_2^4$

$= 12 - 4$

$= 8$