25.3 Area Under a Curve

$$\int_1^5 (2x + 4x^3 + 1)\,dx = \left[x^2 + x^4 + x\right]_1^5$$

↑ "definite integral"

$$= \left[5^2 + 5^4 + 5\right] - \left[1^2 + 1^4 + 1\right]$$

$$= 655 - 3$$

$$= 652$$

Note: Don't write +C for definite integrals

Why not? Would cancel out:

$$\left[x^2 + x^4 + x + C\right]_1^5$$

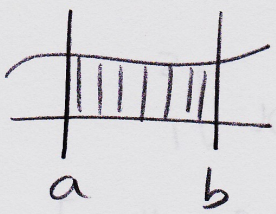
$$= (655 + C) - (3 + C)$$

$$= 652$$

Ex: $\int_2^3 x^3\,dx = \frac{x^4}{4}\Big|_2^3$

$$= \frac{81}{4} - \frac{16}{4}$$

$$= \frac{65}{4}$$

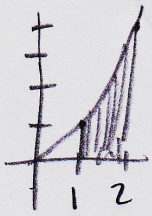


$y = f(x)$

When $f(x) \geq 0$, the area under $y = f(x)$ between $x = a$ and $x = b$ is

$$\int_a^b f(x)\,dx$$

Ex: Find area under $y = x^2$ from $x = 1$ to $x = 2$



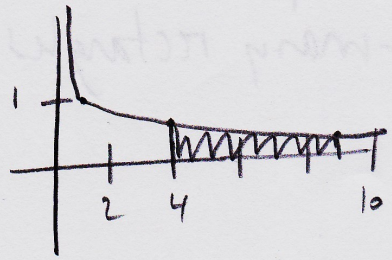
$$A = \int_1^2 x^2\,dx$$

$$= \frac{x^3}{3}\Big|_1^2$$

$$= \frac{8}{3} - \frac{1}{3}$$

$$= \frac{7}{3}$$

Ex: Find area under $y = \frac{1}{\sqrt{x}}$ from $x = 4$ to $x = 9$



$$A = \int_4^9 \frac{1}{\sqrt{x}}\,dx$$

$$= \int_4^9 x^{-1/2}\,dx$$

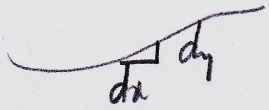
$$= 2x^{1/2}\Big|_4^9$$

$$= 2(3) - 2(2)$$

$$= 2$$

Why does derivative give slope? Integral give area?

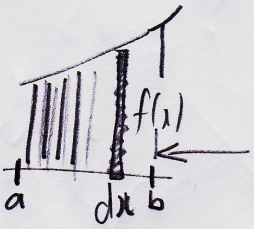
derivative = $\frac{dy}{dx} = \frac{rise}{run} = $ slope



integral = $\int_a^b f(x)dx$

= Sum of tiny areas

= total area



area = $f(x)dx$