25.2 The Indefinite Integral

The integral $\int x^3\,dx = \frac{x^4}{4} + C$

constant

all possible antiderivatives of x^3

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$$

$$\int x^2\,dx = \frac{x^3}{3} + C$$

$$\int 1\,dx = \int x^0\,dx = x + C$$

Ex: Evaluate

a) $\int (5v^3 + 6v)\,dv$

$= \frac{5v^4}{4} + \frac{6v^2}{2} + C$

$= \frac{5v^4}{4} + 3v^2 + C$

b) $\int \left(\frac{1}{r^4} + \frac{1}{r^3} + 8\right) dr$

$= \int \left(r^{-4} + r^{-3} + 8\right) dr$

$= \frac{-r^{-3}}{3} \quad \frac{-r^{-2}}{2} + 8r + C$

c) $\int \left(\sqrt[3]{x} + \frac{1}{x^3}\right) dx$

$= \int \left(x^{1/3} + x^{-3}\right) dx$

$= \frac{3x^{4/3}}{4} \quad -\frac{x^{-2}}{2} + C$

Substitution

Ex: Evaluate $\int x^4(6x^5+13)^2\,dx$

1) Let $u = 6x^5+13$

$du = 30x^4\,dx$

$\frac{du}{30} = x^4\,dx$

2) $\int = \int \frac{u^2\,du}{30}$

$= \frac{1}{30}\frac{u^3}{3} + C$

$= \frac{1}{90}(6x^5+13)^3 + C$

3) original variable

Ex: Evaluate $\int \frac{11x}{\sqrt{x^2+4}}\,dx$

$u = x^2+4$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{11}{2}du = 11x\,dx$

$$\int = \int \frac{11}{2} \frac{1}{\sqrt{u}}\, du$$

$$= \int \frac{11}{2} u^{-1/2}\, du$$

$$= \frac{11}{2}\left(2u^{1/2}\right) + C$$

$$= 11\sqrt{x^2+4} + C$$

Ex: Evaluate $\int \frac{4z-6}{\sqrt{z^2-3z}}\, dz$

$$u = z^2 - 3z$$

$$du = (2z-3)\, dz$$

$$2\,du = (4z-6)\, dz$$

$$\int = \int \frac{2\,du}{\sqrt{u}}$$

$$= \int 2u^{-1/2}\, du$$

$$= 2\left(2u^{1/2}\right) + C$$

$$= 4\sqrt{z^2-3z} + C$$

Ex: Evaluate $\int (2+3x^4)^2 dx$

Expand. No sub required.

$$= \int (4 + 12x^2 + 9x^4) dx$$

$$= 4x + 4x^3 + \frac{9x^5}{5} + C$$

Ex: Evaluate $\int \frac{x^3 + x^2}{\sqrt{x}} dx$

Simplify. No sub required.

$$= \int x^{-1/2} (x^3 + x^2) dx$$

$$= \int (x^{5/2} + x^{3/2}) dx$$

$$= \frac{2}{7} x^{7/2} + \frac{2}{5} x^{5/2} + C$$

Ex: Find y given $\frac{dy}{dx} = 7x^2$ and $(1,3)$ is on the curve

$$\frac{dy}{dx} = 7x^2$$

$$dy = 7x^2\,dx$$

$$\int dy = \int 7x^2\,dx$$

$$\boxed{y = \frac{7x^3}{3} + C}$$

$(x,y) = (1,3)$

$x = 1$
$y = 3$:

$$3 = \frac{7}{3} + C$$

$$C = \frac{2}{3}$$

$$y = \frac{7x^3}{3} + \frac{2}{3}$$

Ex: $f''(x) = 3x + 1$

$f(0) = 2$ and $f(1) = 4$

Find $f(x)$

$$f'(x) = \int f''(x)\,dx$$

$$f'(x) = \int (3x + 1)\,dx$$

$$f'(x) = \frac{3x^2}{2} + x + C$$

$$f(x) = \int f'(x)\,dx$$

$$f(x) = \int \left(\frac{3x^2}{2} + x + C\right) dx$$

$$\boxed{f(x) = \frac{x^3}{2} + \frac{x^2}{2} + Cx + D}$$

new name

$x = 0$, $f(x) = 2$:

$$2 = D$$

$$\boxed{f(x) = \frac{x^3}{2} + \frac{x^2}{2} + Cx + 2}$$

$x = 1$, $f(x) = 4$:

$$4 = \frac{1}{2} + \frac{1}{2} + C + 2$$

$$C = 1$$

$$\boxed{f(x) = \frac{x^3}{2} + \frac{x^2}{2} + x + 2}$$