25.1 Antiderivatives

Find a function $f(x)$ so that

$$f'(x) = 12x^5 \ ?$$

$$f(x) = 2x^6$$

$2x^6$ is an antiderivative of $12x^5$

other possibilities:

$$2x^6 + 1$$

$$2x^6 + \pi$$

$$2x^6 + C \leftarrow \text{any constant}$$

$$\boxed{\text{An antiderivative of } x^n \text{ is } \frac{x^{n+1}}{n+1} \quad (n \neq -1)}$$

Ex: Find an antiderivative

a) $f'(v) = v^3 + v^4$

$$f(v) = \frac{v^4}{4} + \frac{v^5}{5}$$

b) $f'(x) = \sqrt{x} + \frac{1}{x^4} + 3x^7$

$$= x^{1/2} + x^{-4} + 3x^7$$

$$f(x) = \frac{2}{3}x^{3/2} \quad -\frac{1}{3}x^{-3} + \frac{3x^8}{8}$$

c) $f'(x) = 1$

$= x^0$

$f(x) = x$

d) $f'(x) = \frac{1}{\sqrt{x}} + \frac{8}{3}\sqrt[4]{x} + \pi^2$

$= x^{-1/2} + \frac{8}{3}x^{1/4} + \pi^2 \longleftarrow$ constant

$f(x) = 2x^{1/2} + \frac{8}{3}\left(\frac{4}{5}x^{5/4}\right) + \pi^2 x$

$= 2x^{1/2} + \frac{32}{15}x^{5/4} + \pi^2 x$

e) $f'(x) = 8(x^5+2)^7(5x^4)$

Guess and check

$g(x) = (x^5+2)^8$

$g'(x) = 8(x^5+2)^7(5x^4)$ ✓

$$\boxed{f(x) = (x^5+2)^8}$$