RELATED RATES

1. A 40-foot ladder leans against a wall, with its base sliding away from the wall at 2 feet/min. How fast is the height of the ladder changing when the base is 24 feet from the wall?

2. When a spherical balloon is inflated, its radius increases by 3 cm/s. At what rate is the volume changing when the surface area is 100π cm^2?

3. How fast is the slope of the tangent line to $y = \frac{4}{2+5x}$ changing when $x = 2$ if x is increasing by 0.5 units/s?

4. Sand is filled into a large cone with radius 5m and height 10m to form a small cone of sand. If sand is filled at a rate of 10m^3/s, at what rate is the sand's radius increasing when it is 2m?

5. ~~A man~~ 1.8m tall walks with speed ~~2m/s away from~~ a streetlight. If the streetlight sits ~~atop a 6m~~ pole, how fast is the tip of the man's shadow ~~moving~~ along the ground?

OMIT

5. See pages 7 and 8

2.4 Related Rates

Variables x, y, r etc. all depend on time

Ex: $V = \frac{4}{3}\pi r^3$

r depends on t

Find $\frac{dV}{dt}$

$$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt}$$ Chain Rule

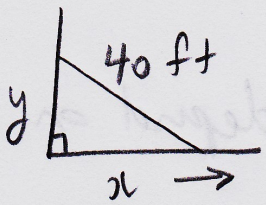
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Ex: x and y depend on t

Find $\frac{d}{dt}[x^2 + y^2]$

$$= 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

(1)



$\frac{dx}{dt} = 2$ ft/min ← rate of change of x w.r.t. time

positive because x is increasing

$\frac{dy}{dt} = ?$ when $x = 24$ ft

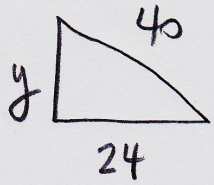
1) Equation

$$x^2 + y^2 = 40^2$$

2) Take $\frac{d}{dt}$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

3) Find any missing values



$$y^2 = 40^2 - 24^2$$

$$y = 32$$

4) Solve

$$2(24)(2) + 2(32)\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -1.5 \text{ ft/min}$$

negative because y is decreasing

(2) $\frac{dr}{dt} = 3 \text{ cm/s}$

$\frac{dV}{dt} = ?$

$SA = 100\pi \text{ cm}^2$

$V_{sphere} = \frac{4}{3}\pi r^3$

$SA_{sphere} = 4\pi r^2$

1) Equation

$V = \frac{4}{3}\pi r^3$

2) Take $\frac{d}{dt}$

$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

3) Missing values?

Given $SA = 100\pi$

$4\pi r^2 = 100\pi$

4) Solve

$\frac{dV}{dt} = 100\pi(3)$

$= 300\pi \text{ cm}^3/\text{s}$

Know all formulas from class:
V, SA of sphere etc.

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(3) y' = slope of tangent line

$\frac{d}{dt}[y'] = ?$ $\quad x = 2$ $\quad \frac{dx}{dt} = 0.5$ units/s

1) Equation

$$y = 4(2+5x)^{-1}$$
$$y' = -4(2+5x)^{-2} \cdot 5$$
$$= -20(2+5x)^{-2}$$

2) Take $\frac{d}{dt}$

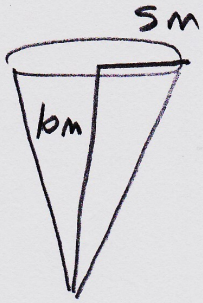
$$\frac{dy'}{dt} = \frac{dy'}{dx}\frac{dx}{dt}$$
$$= y''\frac{dx}{dt}$$
$$= 40(2+5x)^{-3} \cdot 5 \frac{dx}{dt}$$

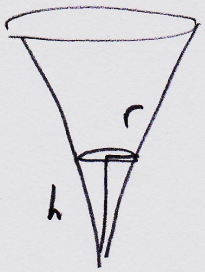
3) Solve

$$\frac{dy'}{dt} = 40(12)^{-3} \cdot 5\,(0.5)$$
$$\approx 0.06 \text{ units/s}$$

Note: slope has no natural units

④





$V_{cone} = \frac{\pi}{3} r^2 h$

$\frac{dV}{dt} = 10 m^3/s$ $\quad \frac{dr}{dt} = ?$ $\quad r = 2m$

1) Equation

$V = \frac{\pi}{3} r^2 h$

Similar triangles to eliminate h

$\frac{10}{5} = \frac{h}{r}$

$h = 2r$

$V = \frac{2\pi}{3} r^3$

2) $\frac{d}{dt}$

$\frac{dV}{dt} = \frac{dV}{dr} \frac{dr}{dt}$

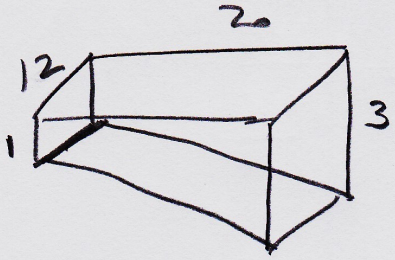
$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$

3) Solve

$10 = 8\pi \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = 0.4 \ m/s$

5

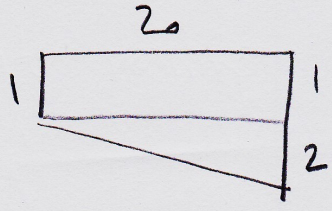
A 12m x 20m swimming pool is filled at a rate of $4\ m^3/min$. Ends are 1m and 3m deep, with a constant slope along the bottom. Rate of change of water depth when depth = 1.5m?



$\frac{dV}{dt} = 4$

Let y = depth

$\frac{dy}{dt} = ?$ $y = 1.5$



Similar triangles

$\frac{20}{2} = \frac{x}{y}$

$x = 10y$

V = triangular area · width of pool

$= \frac{1}{2} xy\,(12)$

$= 6xy$

$= 60y^2$ ← $\boxed{x = 10y}$

$\frac{dV}{dt} = 120y \frac{dy}{dt}$

$4 = 120\,(1.5) \frac{dy}{dt}$

$\frac{dy}{dt} \simeq 0.02$ m/min