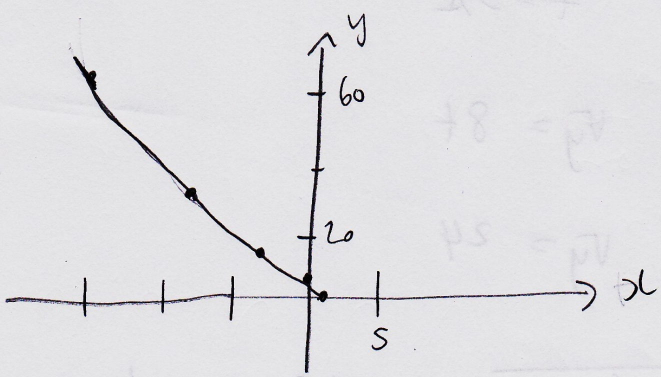
24.3 Curvilinear Motion

Ex: $x = 1 - t^2$ $\quad y = 4t^2$ $\quad$ position (in m)

t : time (in s)

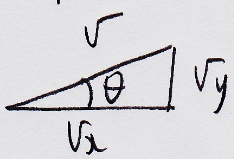
t	x	y
0	1	0
1	0	4
2	-3	16
3	-8	36
4	-15	64
⋮		



Velocity in x-direction $v_x = \frac{dx}{dt}$

" y-direction $v_y = \frac{dy}{dt}$

Velocity represents speed and direction



speed $v = \sqrt{v_x^2 + v_y^2}$

direction $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$ $(+180°?)$

Ex: $x = 1 - t^2$ $y = 4t^2$

Find velocity at $t = 3s$

$v_x = -2t$ $v_y = 8t$

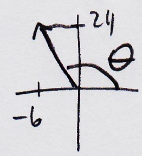
@ $t=3$: $v_x = -6$ $v_y = 24$

speed $v = \sqrt{(-6)^2 + (24)^2} \approx 24.7$ m/s

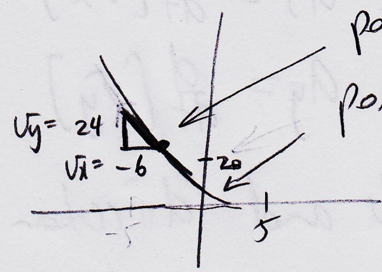
direction $\theta = \tan^{-1}\left(\frac{24}{-6}\right)$ $(+180°?)$

$= -76° + 180°$

$= 104°$



$\boxed{24.7 \text{ m/s at } 104°}$



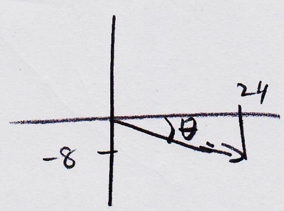
Velocity is tangent to the curve
position curve

Ex: $x = 3t^2$ $\quad y = 1 - t^2$

Find velocity at $t = 4s$

$v_x = 6t$ $\quad v_y = -2t$

@ $t=4$: $\quad v_x = 24$ $\quad v_y = -8$



speed $v = \sqrt{24^2 + (-8)^2}$

≈ 25.3 m/s

direction $\theta = \tan^{-1}\left(\frac{-8}{24}\right)$ $(+180°?)$

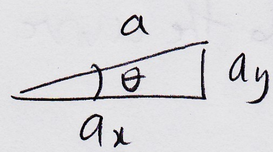
$\approx -18.4°$

$\boxed{25.3 \text{ m/s at } -18.4°}$

Acceleration in x-direction $a_x = \frac{d}{dt}[v_x]$

" y-direction $a_y = \frac{d}{dt}[v_y]$

Acceleration has magnitude and direction



magnitude of acceleration $a = \sqrt{a_x^2 + a_y^2}$

direction $\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$ $(+180°?)$

Ex: $x = 5 + t^2$ $\quad y = 1 + t^3$

Find acceleration at $t = 3s$

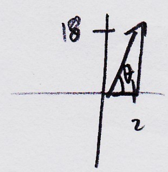
$v_x = 2t$ $\quad v_y = 3t^2$

$a_x = 2$ $\quad a_y = 6t$

@ $t=3$: $a_x = 2$ $\quad a_y = 18$

$a = \sqrt{2^2 + 18^2} \simeq 18.1 \text{ m/s}^2$

$\theta = \tan^{-1}\left(\frac{18}{2}\right)$ $(+180°?)$ $\simeq 83.7°$



$\boxed{18.1 \text{ m/s}^2 \text{ at } 83.7°}$

Recap of Chain Rule

Ex: $y = 100 - 2x + 8x^2$
where x depends on t

Find $\frac{dy}{dt}$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \quad \text{Chain Rule}$$

$$\frac{dy}{dt} = (-2 + 16x) \frac{dx}{dt}$$

Ex: Given position curve

$y = 100 - 0.02x^2$

and $v_x = 9\,m/s$ (constant).

Find velocity at $t = 3s$.

$v_x = 9\,m/s$

v_y ?

$$\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt}$$

$$\frac{dy}{dt} = -0.04x \frac{dx}{dt}$$

$v_y = -0.04x\, v_x$

x? $x =$ initial position + speed $\cdot$ time

0 unless specified

$\frac{m}{s} \cdot s = m$

$x = 0 + v_x t$

@ $t=3$: $x = 9(3) = 27$

$v_y = -0.04(27)(9)$

$= -9.72$

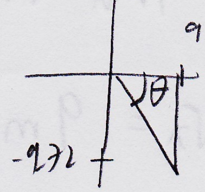
$v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{9^2 + (-9.72)^2}$

≈ 13.2 m/s

$\theta = \tan^{-1}\left(\frac{-9.72}{9}\right)$ $(+180°?)$

$\approx -47.2°$



$\boxed{13.2 \text{ m/s at } -47.2°}$