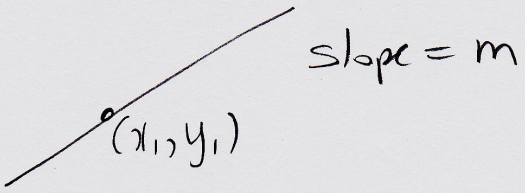
24.1 Tangents and Normals



Point-slope equation of line

$$\boxed{y - y_1 = m(x - x_1)}$$

Ex: Find the equation of the tangent line to $y = x^5 - 4x^3$ at the point $(-1, 3)$

$$y' = 5x^4 - 12x^2$$

$$m_{tan} = y'\Big|_{x=-1} = -7$$

We have $m = -7$ $\quad (x_1, y_1) = (-1, 3)$

$$y - y_1 = m(x - x_1)$$

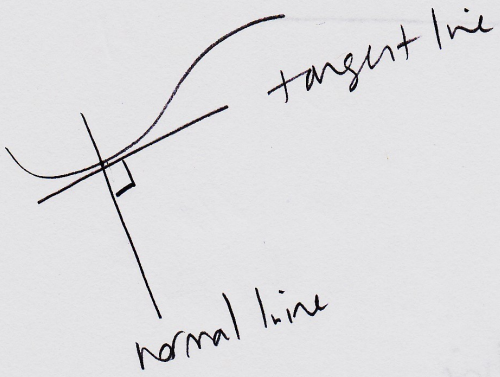
$$y - 3 = -7(x + 1)$$

Text gives answers in standard form

$$ax + by + c = 0$$

$$y - 3 = -7x - 7$$

$$\boxed{7x + y + 4 = 0}$$



Perpendicular lines have slopes that are negative reciprocals

$$m_{normal} = \frac{-1}{m_{tan}}$$

Ex: Find the equation of the normal line to $y = (x^2+1)(4x^3+2)$ at $x = -1$

$$y' = (x^2+1)(12x^2) + (4x^3+2)(2x)$$

$$m_{tan} = y'\Big|_{x=-1} = 2(12) + (-2)(-2)$$
$$= 28$$

$$m_{normal} = \frac{-1}{28}$$

$$x = -1 \Rightarrow y = 2(-2) = -4$$

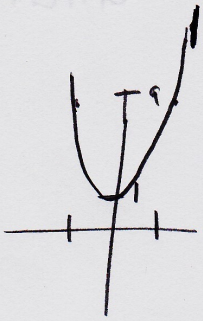
Use $m = \frac{-1}{28}$, $x_1 = -1$, $y_1 = -4$

Normal line $\quad y - y_1 = m(x - x_1)$

$$y + 4 = \frac{-1}{28}(x + 1)$$

$$28y + 112 = -x - 1$$

$$\boxed{x + 28y + 113 = 0}$$



Ex: Given $y = (2x+1)^2$

Find the equation of:

a) the tangent line with slope 20

$$y' = 2(2x+1) \cdot 2$$

$$m_{tan} = 8x + 4$$

Given $m_{tan} = 20$, find (x_1, y_1)

$$8x + 4 = 20$$

$$x = 2$$

$$y = (2x+1)^2 \Rightarrow y = (5)^2 = 25$$

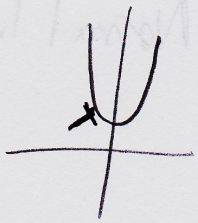
Use $m = 20$, $x_1 = 2$, $y_1 = 25$

$$y - y_1 = m(x - x_1)$$

$$y - 25 = 20(x - 2)$$

$$20x - y - 15 = 0$$

b) the normal line with slope $\frac{1}{4}$



$$m_{tan} = 8x+4$$

$$m_{normal} = \frac{-1}{8x+4}$$

Given $m_{normal} = \frac{1}{4}$, find (x_1, y_1)

$$\frac{-1}{8x+4} = \frac{1}{4}$$

$$-(8x+4) = 4$$

$$-8x-4 = 4$$

$$x = -1$$

$y=(2x+1)^2 \Rightarrow y = (-1)^2 = 1$

Use $m = \frac{1}{4}$ $x_1 = -1$, $y_1 = 1$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{4}(x+1)$$

$$4y - 4 = x + 1$$

$$\boxed{x - 4y + 5 = 0}$$