3.6 Derivatives of Products and Quotients

Product Rule

$(uv)' = uv' + vu'$

or $\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$

Ex: Find y' for $y = (2x+3x^3)(x^2-1)$

$y' = (2x+3x^3)(2x) + (x^2-1)(2+9x^2)$

$= 4x^2 + 6x^4 + 2x^2 + 9x^4 - 2 - 9x^2$

$= 15x^4 - 3x^2 - 2$

Ex: $f = (T^3 + 6T + 7)(T^2 + 8)$

Find $\left.\frac{df}{dT}\right|_{T=-1}$

$\frac{df}{dT} = (T^3+6T+7)(2T) + (T^2+8)(3T^2+6)$

$\left.\frac{df}{dT}\right|_{T=-1} = (0)(-2) + 9(9)$

$= 81$

Quotient Rule

$$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$$

or $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Ex: Find $f'(x)$ and simplify

$$f(x) = \frac{x^2+4}{x-7}$$

$$f'(x) = \frac{(x-7)(2x) - (x^2+4)(1)}{(x-7)^2}$$

Keep denominator factored
Expand numerator

$$= \frac{2x^2 - 14x - x^2 - 4}{(x-7)^2}$$

$$= \frac{x^2 - 14x - 4}{(x-7)^2}$$

Ex: Find $f'(2)$ for $f(x) = \dfrac{(3-7x)(5x+8)}{(4x+1)}$ ← u, ← v

Quotient Rule

$$f'(x) = \frac{(4x+1)\frac{d}{dx}[(3-7x)(5x+8)] - [(3-7x)(5x+8)](4)}{(4x+1)^2}$$

Product Rule

$$= \frac{(4x+1)[(3-7x)(5)+(5x+8)(-7)] - (3-7x)(5x+8)(4)}{(4x+1)^2}$$

$$f'(2) = \frac{9[(-11)(5)+(18)(-7)] - (-11)(18)(4)}{9^2}$$

$$= \frac{-837}{81} \text{ or } \frac{-31}{3}$$

Later on we'll find $f'(x)$ for

$f(x) = \dfrac{e^x \cos x}{x^3+1}$ etc.