23.5 Derivatives of Polynomials

$$\frac{d}{dx}[c] = 0$$

$c \uparrow$ any real #

"Derivative of a constant is zero"

Ex:

$f(x)$	$f'(x)$
7	0
$-4\sqrt{3}$	0
π^3	0

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

$n = 1, 2, 3, \ldots$

Ex:

$f(x)$	$f'(x)$
x	$1x^0 = 1$
x^2	$2x$
x^3	$3x^2$
x^{90}	$90x^{89}$

$$\frac{d}{dx}[cu] = c\frac{du}{dx}$$

c: any real #   v: function of x

Ex: $\frac{d}{dx}[2x^7] = 2\frac{d}{dx}[x^7]$

$$= 2(7x^6)$$
$$= 14x^6$$

$$\frac{d}{dx}[u+v] = \frac{du}{dx} + \frac{dv}{dx}$$

Ex: $\frac{d}{dx}[x^{100} + x^{47}] = 100x^{99} + 47x^{46}$

Ex: Differentiate

a) $f(x) = x^4 + 17\sqrt{3}$ $f'(x) = 4x^3$

b) $f(x) = x^8 - 2x$ $f'(x) = 8x^7 - 2$

c) $f(x) = 12x^6 - 9x + 11$ $f'(x) = 72x^5 - 9$

d) $f(x) = 3x^4 - 7x^3 + 6x^2 + 7$

$$f'(x) = 12x^3 - 21x^2 + 12x$$

Ex: Find slope of tangent line to
$$y = 2x^3 - 5x + 2 \quad \text{at} \quad x = -1$$

slope of tangent line $\quad y' = 6x^2 - 5$

$$y'\Big|_{x=-1} = 1$$

Ex: Where does $f(x) = x^3 - 3x$ have
a horizontal tangent line?

$\underline{\quad\quad}$ slope $= 0$

where is $m_{tan} = 0$?

slope of tangent line $\quad f'(x) = 3x^2 - 3$

$$\text{Set } 3x^2 - 3 = 0$$
$$3(x^2 - 1) = 0$$
$$3(x-1)(x+1) = 0$$

x-values $\qquad\qquad x = \pm 1$

points $\qquad (-1, 2) \qquad$ and $\quad (1, -2)$

$\qquad\qquad\qquad\quad \uparrow \qquad\qquad\qquad\qquad\quad \uparrow$

$\qquad\qquad\qquad f(-1) \qquad\qquad\qquad\qquad f(1)$

$y = x^3 - 3x$
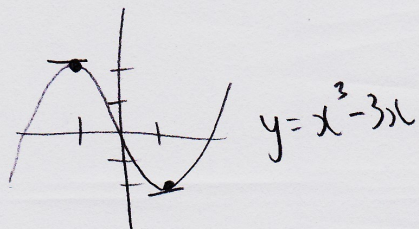
Ex: Find the derivative

a) $f = \pi r + 7r^{10}$

$\dfrac{df}{dr}$ or $f' = \pi + 70r^9$

b) $V = 12m^3 + 8m^2$

$\dfrac{dV}{dm}$ or $V' = 36m^2 - 16m$

Ex: Find $\dfrac{d}{dt}\left[t^6 + t^5 + t^4\sqrt{r} + rt + r^5 \right]$

when r is constant

$= 6t^5 + 5t^4 + 4t^3(\sqrt{r}) + r + 0$

$= 6t^5 + 5t^4 + 4\sqrt{r}\,t^3 + r$