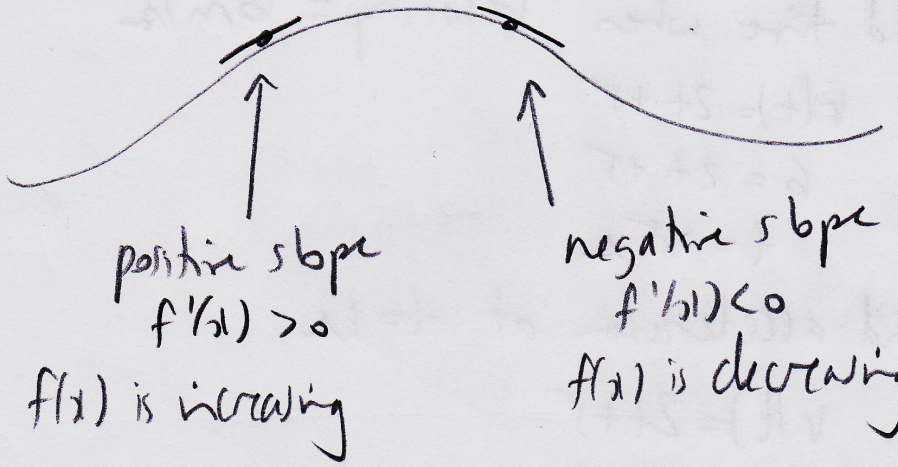
# 23.4 Instantaneous Rate of Change and 23.9 Higher Derivatives

$f'(x)$ represents: $m_{tan}$

instantaneous rate of change of $f(x)$

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Motion in a straight line e.g. car in driveway

$s(t)$: displacement after $t$ seconds (m)

$v(t)$: velocity (m/s)

$a(t)$: acceleration ($m/s^2$)

$s(t)$

$v(t) = s'(t)$

$a(t) = v'(t)$

"Velocity is the instantaneous rate of change of displacement with respect to time"

**Ex:** $s(t) = t^2 + 5t$

$s$: in m  $t$: in s

a) Find velocity at $t = 1$ s

$$v(t) = s'(t)$$
$$= 2t + 5$$
$$v(1) = 7 \text{ m/s}$$

b) Find time when velocity = 6 m/s

$$v(t) = 2t + 5$$
$$6 = 2t + 5$$
$$t = 0{,}5 \text{ s}$$

c) Find acceleration at $t = 1$ s

$$v(t) = 2t + 5$$
$$a(t) = v'(t)$$
$$= 2$$
$$a(1) = 2 \text{ m/s}^2$$

Ex: Find all derivatives of $f(x) = x^4 + 7x^2$

$f'(x) = 4x^3 + 14x$

$f''(x) = 12x^2$ "second derivative"

$f'''(x) \sim f^{(3)}(x) = 24x$

$f^{(4)}(x) = 24$

$f^{(5)}(x) = 0$ etc.

$s(t)$

$v(t) = s'(t)$

$a(t) = v'(t) = s''(t)$

Ex: $s(t) = 8t^3 - 12t^2 + 5$

Find velocity and acceleration

$v(t) = 24t^2 - 24t$

$a(t) = 48t - 24$

$$v = \frac{ds}{dt}$$

Velocity is the rate of change of displacement, with respect to time.

Units: $\frac{\text{Units of } s}{\text{unit of } t}$

$= m/s$

Ex: A circular oil spill has radius $r$ and area $A$. Find the (instantaneous) rate of change of $A$ with respect to $r$.

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

rate of change of $A$ w.r.t. $r$

units: $cm^2/cm$

$\frac{\text{units of } A}{\text{units of } r}$

**Ex:** A cylinder is heated and expands such that its height is always 3 times its radius.

Find (instantaneous) rate of change of volume with respect to radius when radius = 10 cm.

$V = \pi r^2 h$

$\boxed{h = 3r}$

$V = 3\pi r^3$

$\frac{dV}{dr} = 9\pi r^2$ $\quad cm^3/cm$

↑ units of $V$ / units of $r$

rate of change of $V$ w.r.t. $r$

$$\left.\frac{dV}{dr}\right|_{r=10cm} = 900\pi \ cm^3/cm$$