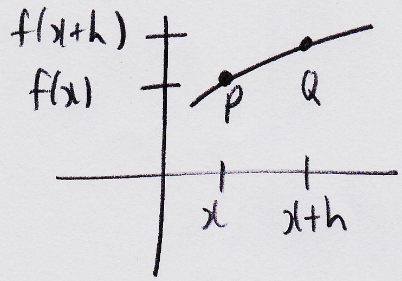
23.2 The Slope of a Tangent Line

Tangent line to a curve: touches curve at a single point

Slope of tangent line is written m_{tan}

Consider a short line segment PQ
Let h = small positive # e.g. $h = 0.1$



slope of PQ $\quad m_{PQ} = \dfrac{\text{rise}}{\text{run}}$

$$= \frac{f(x+h) - f(x)}{x+h-x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

As $h \to 0$, $m_{PQ} \to m_{tan}$

$$\boxed{m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}} \quad \bigstar$$

$\underline{Ex}$: Find m_{tan} to $y = x^2 + 5$ at the point $(x, y) = (1, 6)$

$$m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$x = 1$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$f(x) = x^2 + 5$

$$= \lim_{h \to 0} \frac{(1+h)^2 + 5 - \boxed{6}}{h}$$

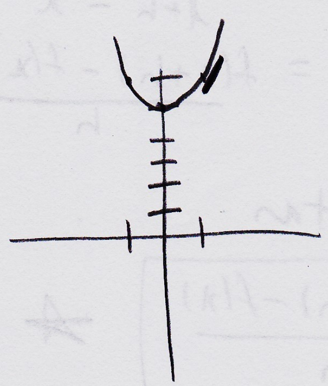
$\left(\frac{0}{0}\right)$

$$= \lim_{h \to 0} \frac{1 + 2h + h^2 + 5 - 6}{h}$$

$$= \lim_{h \to 0} \frac{2h + h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2+h)}{\cancel{h}}$$

$$= 2$$



Ex: Find m_{tan} to $y = x^2 + 4x$

$$m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 4(x+h) - [x^2 + 4x]}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 4x + 4h - x^2 - 4x}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 4h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 4)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h + 4$$

$$= 2x + 4$$

When $x = 0$, $m_{tan} = 4$

$x = -1$, $m_{tan} = 2$

$x = 7$ $m_{tan} = 18$

Ex: Find m_{tan} to $y = 2x^2 - 6x + 3$

$$m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad \left(\frac{0}{0}\right)$$

$$= \lim_{h \to 0} \frac{\left[2(x+h)^2 - 6(x+h) + 3\right] - \left[2x^2 - 6x + 3\right]}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) \cancel{-6x} - 6h \cancel{+3} - 2x^2 \cancel{+6x} \cancel{-3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - 6h \cancel{-2x^2}}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 - 6h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 6)}{\cancel{h}}$$

$$= 4x - 6$$