Big ideas in Calculus:

1) Rates of change : velocity, acceleration etc.

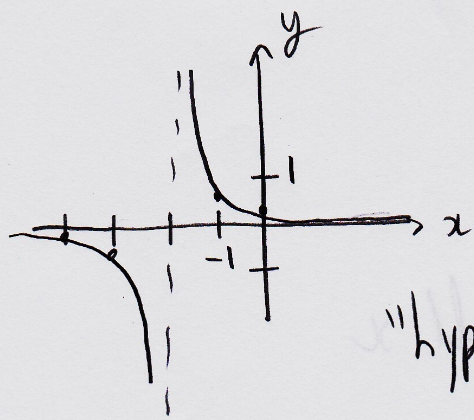
2) Areas, Volumes, Centre of Mass etc.

23.1 Limits

A function is <u>continuous</u> at an x-value if there is no jump or hole there.

<u>Ex</u>: $f(x) = \frac{1}{2x+4}$

x	$f(x)$
-4	$-1/4$
-3	$-1/2$
-2	undefined
-1	$1/2$
0	$1/4$



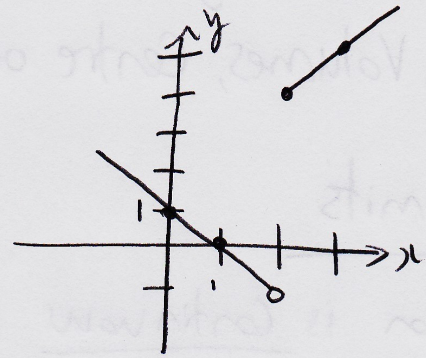
"hyperbola"

$f(x)$ is continuous for $x \neq -2$

$x =$ all real #, except $x = -2$

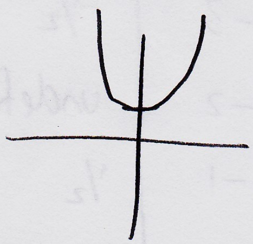
Ex: $f(x) = \begin{cases} x+2, & x \geq 2 \\ 1-x, & x < 2 \end{cases}$

	x	$f(x)$
$x<2$	0	1
	1	0
$x \geq 2$	2	4
	3	5



$f(x)$ is continuous for $x \neq 2$

Ex: $f(x) = x^2+1$ parabola



$f(x)$ is continuous for all x

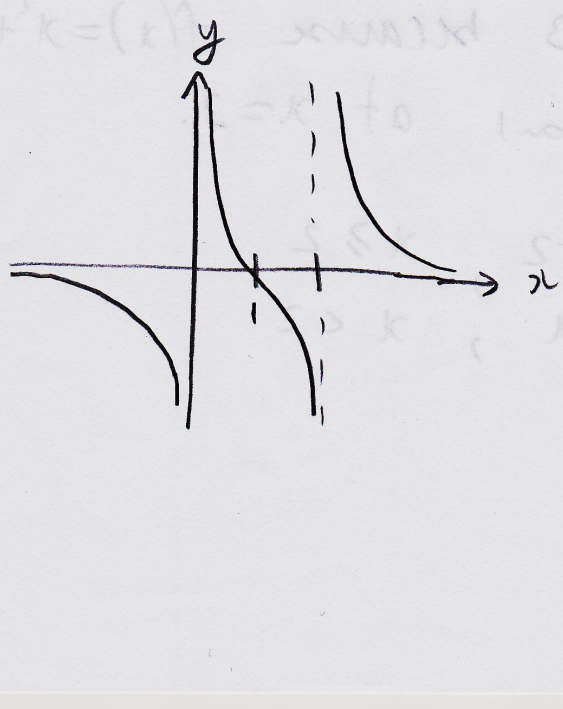
Ex: $f(x) = \dfrac{x-1}{x^2-2x}$

$= \dfrac{x-1}{x(x-2)}$

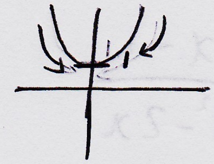
undefined when $x = 0, 2$

$f(x)$ is continuous for $x \neq 0, 2$

Quick way to graph: Google Wolfram Alpha "graph y= (x-1)/(x^2 -2x)" Click $\boxed{=}$



Ex: $f(x) = x^2+1$



As $x \to 0$ from the left, $f(x) \to 1$

" right, $f(x) \to 1$

We write: $\lim_{x \to 0} f(x) = 1$

"The limit of $f(x)$ as x approaches 0 is 1"

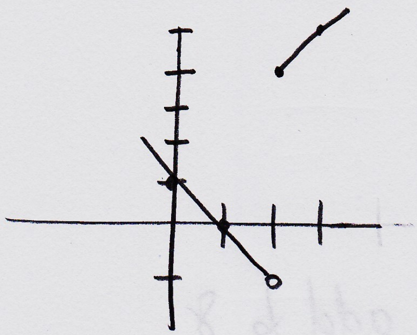
Fact: If $f(x)$ is continuous at $x=a$ then $\lim_{x \to a} f(x) = f(a)$

Ex: $\lim_{x \to 3} x^2+1 = 3^2+1$

$= 10$

Can plug in $x=3$ because $f(x) = x^2+1$ is continuous at $x=3$.

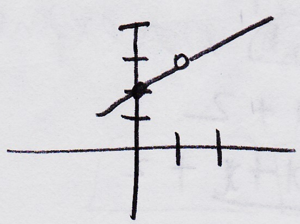
Ex: $f(x) = \begin{cases} x+2, & x \geq 2 \\ 1-x, & x < 2 \end{cases}$



As $x \to 2$ from the left, $f(x) \to -1$
" right, $f(x) \to 4$ } don't agree

$\lim_{x \to 2} f(x)$ does not exist
(d.n.e.)

<u>Ex</u>: $f(x) = x+2,\ x \neq 1$



$\lim_{x \to 1} f(x) = 3$

Limit exists even though $f(x)$ is not defined at $x=1$

Review: Factoring

i) x^2+8x+7

$= (x+7)(x+1)$

add to 8

multiply to 7

ii) x^2-100

$= (x-10)(x+10)$

iii) $2x^2+5x+2$

ac method

add to 5

multiply to $ac = 4$

4, 1

$2x^2+5x+2$

$= \boxed{2x^2+4x}+\boxed{x+2}$

$= 2x(x+2)+1(x+2)$

$= (2x+1)(x+2)$

Ex: Find $\lim_{x \to 5} \frac{x^2-25}{x-5}$ ← not continuous at $x=5$

get $\frac{0}{0}$ when we sub

$$= \lim_{x \to 5} \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}1} \leftarrow \text{continuous at } x=5$$

$$= \frac{10}{1}$$

$$= 10$$

Ex: $\lim_{x \to -4} \frac{x^2-3x-28}{2x+8}$ $\left(\frac{0}{0}\right)$

$$= \lim_{x \to -4} \frac{(x-7)\cancel{(x+4)}}{2\cancel{(x+4)}}$$

$$= \frac{-11}{2}$$

Ex: $\lim_{x \to 0} \frac{x^2+9x}{x^2+x}$ $\left(\frac{0}{0}\right)$

$$= \lim_{x \to 0} \frac{\cancel{x}(x+9)}{\cancel{x}(x+1)}$$

$$= \frac{9}{1}$$

$$= 9$$

Ex: $\lim_{x \to 2} \frac{x^2+24}{x+11}$ ← continuous at $x=2$

can sub in

$= \frac{28}{13}$

Ex: $\lim_{x \to 8} \frac{1}{x-8}$

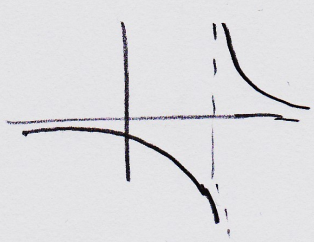
Can't factor.
Sketch or table of values.

x	7.9	7.99	8.01	8.1
$f(x) = \frac{1}{x-8}$	-10	-100	100	10

As $x \to 8$ from the left, $f(x) \to -\infty$

" right, $f(x) \to \infty$

$\lim_{x \to 8} \frac{1}{x-8}$ d.n.e.



To evaluate limits

Sub in if function is continuous at the x-value

Factor

Last resort: table of values or sketch