2.3 Logical Equivalence

Let 0 = false
 1 = true

Ex: Build the truth table for $p \wedge q$

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Interpretation:

1st row says when p and q are both false, $p \wedge q$ is false.

2nd row says when p is false and q is true, $p \wedge q$ is false.

⋮

4th row says when p and q are both true, $p \wedge q$ is true.

Ex: Build the truth table for $\sim(p \vee q)$

p	q	$p \vee q$	$\sim(p \vee q)$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

If we have 3 propositions p, q, r then we need 8 rows:

p	q	r
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Ex: Build the truth table for

$\sim(p \vee \sim q) \wedge \sim r$

p	q	r	$\sim q$	$p \vee \sim q$	$\sim(p \vee \sim q)$	$\sim r$	$\sim(p \vee \sim q) \wedge \sim r$
0	0	0	1	1	0	1	0
0	0	1	1	1	0	0	0
0	1	0	0	0	1	1	1
0	1	1	0	0	1	0	0
1	0	0	1	1	0	1	0
1	0	1	1	1	0	0	0
1	1	0	0	1	0	1	0
1	1	1	0	1	0	0	0

Two logical expressions are logically equivalent if they have the same sequence of truth values.

Ex: Is $\sim(p \vee q)$ logically equivalent to $\sim p \wedge \sim q$?

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

identical

Yes. We write $\sim(p \vee q) \Longleftrightarrow \sim p \wedge \sim q$

Ex: Is $\sim(p \oplus q)$ logically equivalent to $\sim p \oplus \sim q$?

p	q	$p \oplus q$	$\sim(p \oplus q)$	$\sim p$	$\sim q$	$\sim p \oplus \sim q$
0	0	0	1	1	1	0
0	1	1	0	1	0	1
1	0	1	0	0	1	1
1	1	0	1	0	0	0

Not identical

No. We write $\sim(p \oplus q) \not\Leftrightarrow \sim p \oplus \sim q$

Notation:

0 is the expression that is always false.
1 " " true.

Ex: Simplify $p \wedge \sim p$ using a truth table.

p	$\sim p$	$p \wedge \sim p$
0	1	0
1	0	0

$p \wedge \sim p \Leftrightarrow 0$

Ex: Simplify $p \wedge 1$ using a truth table.

p	1	$p \wedge 1$
0	1	0
1	1	1

Identical

$$p \wedge 1 \iff p$$

Ex: Simplify $(\sim p \wedge \sim q) \vee (p \wedge \sim q)$ using a truth table.

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \wedge \sim q$	$(\sim p \wedge \sim q) \vee (p \wedge \sim q)$
0	0	1	1	1	0	1
0	1	1	0	0	0	0
1	0	0	1	0	1	1
1	1	0	0	0	0	0

Identical

$$(\sim p \wedge \sim q) \vee (p \wedge \sim q) \iff \sim q$$