

2.1 Intro to Logic

Logical proposition: A statement that is true or false.

Examples of logical propositions:

Python is a coding language. (TRUE)

7 is an even integer. (FALSE)

Not logical propositions:

Put your books away.

Where is Saryta's office?

(He) is a programmer.

→
undefined

We use symbols p, q, r, s, t, \dots
for logical propositions.

Ex: Let $p =$ "Leah drinks coffee."

$\sim p$ is the negation of p .
Pronounced "not p ."

$\sim p$ is TRUE when p is FALSE.
 $\sim p$ is FALSE when p is TRUE.

Ex: p = "I have at most two siblings."
 $\sim p$ = "I have more than two siblings."

Ex: Write the negation of:

a) p = "All of us are going to study."

$\sim p$ = "At least one of us is not going to study."

b) q = "None of us are going to study."

$\sim q$ = "At least one of us is going to study."

c) r = "My Visa balance is positive."

$\sim r$ = "My Visa balance is not positive."
(The balance could be 0 or negative.)

$p \wedge q$: p and q

$p \wedge q$ is TRUE when both p and q are TRUE.

$p \wedge q$ is FALSE otherwise.

$p \vee q$: p or q

$p \vee q$ is TRUE when at least one of p or q is TRUE.

$p \vee q$ is FALSE otherwise.

$p \oplus q$: p exclusive or q

$p \oplus q$ is TRUE when exactly one of p or q is TRUE.

otherwise $p \oplus q$ is FALSE.

Ex: p is FALSE and q is TRUE. State the truth value of :

- a) $p \vee q$ TRUE
- b) $p \oplus q$ TRUE
- c) $p \wedge q$ FALSE

Ex: Is the "or" exclusive or inclusive?

a) Would you like milk or sugar in your coffee?
inclusive

b) Are you coming to the party or not?
exclusive

Order of Operations

\sim is done first

then \wedge

then \vee

Brackets override the order

$p \vee q \wedge r$ is the same as $p \vee (q \wedge r)$

$\sim p \vee q$ " $(\sim p) \vee q$

negation of $p \vee q$ " $\sim(p \vee q)$

$p \vee \sim q \wedge r$ " $p \vee ((\sim q) \wedge r)$

Ex: I drank coffee today.

Did I drink tea or coffee today? YES

Ex: I drank tea or coffee today.

Did I drink coffee today? MAYBE

Ex:

p = "Pumpkins are a fruit."
 q = "Quarters are round."
Translate to logic.

a) Either pumpkins are a fruit or quarters are round.
 $p \vee q$

b) Either pumpkins are a fruit or quarters are round,
but not both.
 $p \oplus q$

c) Pumpkins aren't a fruit or quarters aren't round.
 $\sim p \vee \sim q$

d) It is not true that pumpkins are a fruit
or quarters are round.
 $\sim (p \vee q)$

e) It is not true that pumpkins aren't a fruit.
 $\sim (\sim p)$

Translate to English

f)

$p \wedge \sim q$

Pumpkins are a fruit and quarters aren't round.

Alternatively:

Pumpkins are a fruit ~~but~~ quarters aren't round.
↙
and

g)

$q \oplus \sim p$

Quarters are round or pumpkins aren't a fruit,
but not both.

h)

$(q \wedge \sim p) \vee \sim q$

Either quarters are round and pumpkins aren't a fruit or quarters aren't round.