

$$(17) \quad (p \wedge (p \wedge q)) \vee (q \wedge \sim q)$$

$$\Leftrightarrow ((p \wedge p) \wedge q) \vee (q \wedge \sim q) \quad \text{Associative}$$

$$\Leftrightarrow (p \wedge q) \vee (q \wedge \sim q) \quad \text{Idempotent}$$

$$\Leftrightarrow (p \wedge q) \vee 0 \quad \text{Complement}$$

$$\Leftrightarrow p \wedge q \quad \text{Identity}$$

$$(18) \quad A \bar{B} (B + C + \overline{B + C}) + \bar{B} A$$

$$\Leftrightarrow A \bar{B} (1) + \bar{B} A \quad \text{Complement}$$

$$\Leftrightarrow A \bar{B} + \bar{B} A \quad \text{Identity}$$

$$\Leftrightarrow \bar{B} A + \bar{B} A \quad \text{Commutative}$$

$$\Leftrightarrow \bar{B} A \quad \text{Idempotent}$$

$$(19) (\overline{B+A} + \overline{B}A)(B + \overline{A})$$

$$\Leftrightarrow (\overline{B}\overline{A} + \overline{B}A)(B + \overline{A})$$

De Morgan's

Distributive

Complement

Identity

Absorption

$$\Leftrightarrow \overline{B}(\overline{A} + A)(B + \overline{A})$$

$$\Leftrightarrow \overline{B}(1)(B + \overline{A})$$

$$\Leftrightarrow \overline{B}(B + \overline{A})$$

$$\Leftrightarrow \overline{B}\overline{A}$$

(20)

$$\sim(\sim p \wedge \sim q) \wedge (p \wedge \sim r)$$

$$\Leftrightarrow (\sim\sim p \vee \sim\sim q) \wedge (p \wedge \sim r)$$

De Morgan's

$$\Leftrightarrow (p \vee q) \wedge (p \wedge \sim r)$$

Complement

$$\Leftrightarrow ((p \vee q) \wedge p) \wedge \sim r$$

Associative

$$\Leftrightarrow (p \wedge (p \vee q)) \wedge \sim r$$

Commutative

$$\Leftrightarrow p \wedge \sim r$$

Absorption

(21)

- a) MAYBE
- b) No
- c) YES
- d) MAYBE

(22)

- a) Contrapositive of original:  
Logically equivalent to original.
- b) Converse of original.  
Not logically equivalent to original.
- c) Inverse of original.  
Not logically equivalent to original.

(23)

a) No

b) YES

c) No

d) YES

(24)

a)  $a_n = \sqrt{n}$  for  $2 \leq n \leq 7$

Alternatively:

$a_n = \sqrt{n+1}$  for  $1 \leq n \leq 6$

b) Each term is the previous term + 2

$$\begin{cases} a_1 = 6 \\ a_n = a_{n-1} + 2 \quad \text{for } n \geq 2 \end{cases}$$

Alternatively:

$$\begin{cases} a_0 = 6 \\ a_n = a_{n-1} + 2 \quad \text{for } n \geq 1 \end{cases}$$

(25)

$$\sum_{i=3}^8 (2^i - 3)$$

$$= (2^3 - 3) + (2^4 - 3) + \dots + (2^8 - 3)$$

$$= 5 + 13 + 29 + 61 + 125 + 253$$

$$= 486$$

(26)

Arithmetic sequence

with  $a_1 = -38$  and  $d = -3$

$$a_n = a_m + (n-m)d \quad \text{for } n \geq m$$

Sub  $m=1$  :  $a_n = a_1 + (n-1)d \quad \text{for } n \geq 1$

$$a_n = -38 + (n-1)(-3) \quad \text{"}$$

$$a_n = -38 - 3n + 3 \quad \text{"}$$

$$a_n = -35 - 3n \quad \text{for } n \geq 1$$

$$(27) \quad a_n = a_m + (n-m)d \quad \text{for } n \geq m$$

Find  $d$ :

Sub  $n=21$  and  $m=8$ :

$$a_{21} = a_8 + 13d$$

$$-67 = -15 + 13d$$

$$-52 = 13d$$

$$d = -4$$

Find  $a_1$ :

$$a_n = a_m + (n-m)d \quad \text{for } n \geq m$$

Sub  $n=8$  and  $m=1$ :

$$a_8 = a_1 + 7d$$

$$-15 = a_1 + 7(-4)$$

$$-15 = a_1 - 28$$

$$13 = a_1$$

(28)

$$\sum_{i=3}^{43} (5i+2)$$

$$= 17 + 22 + 27 + \dots + 217$$

Arithmetic Series

$a_m$   $a_n$

$$\begin{aligned}
 k &= \# \text{ of terms} \\
 &= n - m + 1 \\
 &= 43 - 3 + 1 \\
 &= 41
 \end{aligned}$$

$$\begin{aligned}
 S_k &= \frac{k}{2} (a_m + a_n) \\
 &= \frac{41}{2} (17 + 217) \\
 &= 4797
 \end{aligned}$$

(29)

Geometric Sequence  
with  $a_1 = 81$   $r = \frac{1}{3}$

$$\begin{cases} a_1 = 81 \\ a_n = \frac{1}{3} a_{n-1}, \quad n \geq 2 \end{cases}$$

30 Geometric Sequence  
with  $a_1 = 4$   $r = -2$

$$a_n = a_m r^{n-m} \quad \text{for } n \geq m$$

Sub  $n = 20, m = 1$ :

$$\begin{aligned} a_{20} &= a_1 r^{19} \\ &= 4(-2)^{19} \\ &= -2097152 \end{aligned}$$

31 Geometric series with  
 $a_1 = 1000$   $r = \frac{1}{2}$

$$a) \quad S_k = \frac{a_m (1 - r^k)}{1 - r}$$

Sub  $k = 12, m = 1$ :

$$\begin{aligned} S_{12} &= \frac{a_1 (1 - r^{12})}{1 - r} \\ &= \frac{1000 \left(1 - \left(\frac{1}{2}\right)^{12}\right)}{\left(\frac{1}{2}\right)} \\ &\approx 1999.51 \end{aligned}$$



b)  $-1 < r < 1$  ✓

$$\begin{aligned} S_{\infty} &= \frac{a_n}{1-r} \\ &= \frac{a_1}{1-r} \\ &= \frac{1000}{\left(\frac{1}{2}\right)} \\ &= 2000 \end{aligned}$$

32

- a) Program 2
- b) Program 1
- c) 100,000

33

- a)  $O(n^2)$
- b)  $O(1)$
- c)  $O(n)$
- d)  $O(2^n)$
- e)  $O(n!)$
- f)  $O(n!)$
- g)  $O(n \log n)$
- h)  $O(\log n)$
- i)  $O(n)$

34

- $O(1)$ ,  $O(\log n)$ ,  $O(n)$ ,  $O(n \log n)$ ,
- $O(n^2)$ ,  $O(2^n)$ ,  $O(n!)$

35

a) Each car is an experimental unit.

b) Make QUALITATIVE

Distance on Odometer DISCRETE

Diameter of Tire CONTINUOUS

36

a) Unimodal Skewed Left

b) The grades in the 0-10% range are outliers.

c) The 70-80% range is the mode.

d)  $\frac{10}{100} = 10\%$

37

The percentages do not add to 100%.